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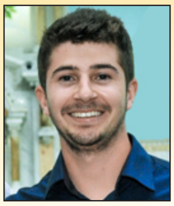
Reliability of internal column-slab connection under punching according to NBR 6118:2014

Confiabilidade da ligação laje-pilar interno sob punção de acordo com a NBR 6118:2014



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Abstract

This article presents a study on the reliability of internal column-slab connection under punching, designed according to the Brazilian Standard NBR 6118:2014. The evaluation of reliability was made by comparing the reliability index β with the target reliability index recommended by the fib Model Code 2010. The reliability indexes were calculated through statistical analysis of the data obtained with numerical simulations using the Monte Carlo method, with Latin Hypercube sample, through ANSYS software. The results showed that, for most slabs, the indexes of reliability β presented satisfactory results. However, some of the tested slabs presented results below the assumed limits. Therefore, this article suggests that the Brazilian Standard NBR 6118:2014 is appropriate for most flat slabs without shear reinforcement.

Keywords: structural reliability, flat slabs, punching, reinforced concrete.

Resumo

Este artigo apresenta um estudo sobre a confiabilidade da ligação laje-pilar interno de lajes lisas, sem armadura de cisalhamento, sob punção, dimensionadas de acordo com a norma brasileira NBR 6118:2014. A avaliação da confiabilidade foi feita através da comparação do índice de confiabilidade β com o índice de confiabilidade alvo recomendado pelo Código Modelo 2010 da fib. Os índices de confiabilidade foram calculados a partir da análise estatística dos dados obtidos com simulações numéricas utilizando o Método de Monte Carlo, com amostragem por Latin Hypercube, através do software ANSYS. Os resultados encontrados mostram que, para a maior parte das lajes analisadas nesse trabalho, os índices de confiabilidade β apresentam resultados satisfatórios. Entretanto, algumas das lajes analisadas apresentam resultados abaixo dos limites estabelecidos. Assim, o estudo indica que a NBR 6118:2014 está adequada para a maioria das lajes lisas sem armadura de cisalhamento.

Palavras-chave: confiabilidade estrutural, lajes lisas, punção, concreto armado.

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1. Introduction

Flat slab is a concrete slab supported directly by columns. This type of slab has several advantages in comparison to slabs supported on beams, like reduction of floor-to-floor height, simplified and diminished formwork, shorter construction time and more architectural freedom. Simple flat slabs are more usual in civil construction than flat slabs with drop panels or column heads due to easiness in the formwork erection. Some disadvantages must be considered, like the possibility of failure by punching.

The punching shear is defined as a failure mechanism by shear under the action of concentrated loads. These concentrated loads create punching shear in the region close to the loading point, and can lead to a sudden failure of the structure without previous notice. This type of failure can be a catastrophic situation in civil construction, since it does not allow the professionals or the user to perceive the imminent collapse. In addition, this rupture can promote the progressive collapse of the structure, causing great economic and human losses. Therefore, the evaluation of safety in designs involving flat slabs is of fundamental importance for the efficient and reliable use of slabs.

The structure safety can be determined through the Reliability Theory. Through the concepts of this theory and Monte Carlo simulation using the ANSYS program [1], the reliability of flat slabs designed according to the calculation model proposed by the Brazilian Standard for concrete design NBR 6118: 2014 [2] is evaluated.

1.1 Reliability analyses

The design of structural systems is based on the comparison between loading and the resistance of the system. As these parameters are obtained from random variables, the behavior of the structure is also a random phenomenon (Ang and Tang [3]). Therefore, a probabilistic analysis of the structure behavior is necessary to understand and design structures with adequate levels of safety. The structure behavior is affected by the basic random variables behavior R = (r1, r2, ..., rn) and S = (s1, s2, ..., sn), which are related to the resistance and the load, respectively. Assuming the knowledge of the probability distributions of the resistance (R) and the load (S), the probability of failure of the structure is given by expression (1).

$$P_f = P(R < S) = \sum_{\forall S} P(R < S|S = s).P(S = s) \tag{1}$$

Assuming the resistance and loading as a function of random variables that is statistically independent, it can be written in the form of the following equation (2):

$$P_f = P(R < S) = \sum_{\forall S} P(R < S).P(S = s) \tag{2}$$

For continuous random variables:

$$P(R < S) = \mathrm{F}_R(s) \tag{3}$$

$$P(s \leq S \leq s + ds) = f_S(s)ds \tag{4}$$

where:

Fr: cumulative distribution function of resistance;

fs: probability density function of load.

Thus, the probability of failure in continuous random and statistically independent variables is calculated through the basic equation of the risk-based design concept, which can be written in the form of the equation (5):

$$P_f = \int_{-\infty}^{+\infty} \mathrm{F}_R(s) f_S(s) ds \tag{5}$$

When dealing with functions of complex random variables or when the probability distributions of these functions are not known, the evaluation of integral (5) is often impossible to obtain. In these cases, one of the alternatives for determining the probability of failure is through the Monte Carlo simulation.

1.2 Monte Carlo simulation

A user can perform simulation methods in reliability analysis without advanced knowledge in probability and statistics. The advantages of these methods are notorious when working with complex random variables functions or when the behavior of the resistance or load functions is not known.

Monte Carlo methods rely on a repeated process, simulating the response of random variables functions, using deterministic values of the variables in each simulation cycle. The deterministic values are stipulated based on the probability distributions of the random variables. The repetition of the process generates a sample of solutions, each corresponding to a different set of values of the random variables (Real [4]). A sample obtained via Monte Carlo methods is similar to a sample of experimental observations.

This method presents good precision, but the method convergence requires that a lot of simulations are generated, and thus the method requires a great computational effort to perform complex analysis.

In this study, the Monte Carlo method was used to obtain a sufficient data set to statistically evaluate the functions of R and S random variables. Using this data set, statistical tests such as Kolmogorov-Smirnov test [3] were performed, in order to define the probability distributions of the R and S random variable functions. Thus, the probability of failure was determined by the numerical calculation of the expression (5).

1.3 Reliability index – β

The safety level associated with a structure can be represented by reliability index β. The higher the value of this index is, the lower the probability of the system failure.

The reliability index β can be defined by the expression (6) [5]:

$$\beta = -\Phi^{-1}(P_f) \tag{6}$$

where:

ϕ= standard normal distribution

Pf= probability of failure of the system

The reliability index β is used by standards and codes to determine the level of safety that a structure must achieve. The choice for this index should reflect the consequences of the failure of the

Table 1
Reliability target index [6]

Relative cost of safety measure	Expect consequences given a failure			
	Low	Some	Moderate	High
High	0.0	1.5	2.3	3.1
Moderate	1.3	2.3	3.1	3.8
Low	2.3	3.1	3.8	4.3

structure. The reliability index that a structure must achieve is called target reliability index. In this study, the target reliability index presented in the fib Model Code 2010 [6] was adopted. This reliability index to be achieved by a given structure in a period of 50 years considers the consequence of the structure failure and the relative cost of the safety measures, as shown in Table 1.

2. Methodology

Fifty-four flat slabs without shear reinforcement were designed according to NBR 6118:2014 in order to verify the reliability of the slab-central column intersection. The study on the reliability of these flat slabs under punching shear was developed by comparing the load-carrying capacities of the slab-column intersection (kN), R, and the total load of the slab (kN), S. The evaluated parameters in this study were the concrete characteristic compressive strength, the slab thickness and the live load.
Three characteristic compressive strengths of 30, 60 and 90 MPa, three values for slab thickness, 16, 20 and 24 cm, and three values of live loads, 2, 4 and 6.0 kN / m² were selected, composing a set of 27 slabs. In addition, the slabs internal forces and moments were calculated for the design of the flexural reinforcement by the Approximate Elastic Process (AEP) and by the Finite Element Method (FEM), thus composing a total of 54 slabs
The set of slabs were denominated by the letter L and 5 numbers. The first and second digits represent the characteristic compressive strength of the concrete, in MPa. The third digit represents the live load, in kN / m², and the fourth and fifth the thickness of the slab, in cm.
In order to verify the reliability of the slab-column intersection, a slab of 11 x 11 m, supported by 40 x 40 cm columns spaced at 4.5 m was analyzed, reproducing the situation of a floor with orthogonally distributed and uniformly spaced columns, shown in Figure 1. The functions of random variables of the resistance and the loading of these slabs were adopted as the failure load of the slabs and the total external load applied to the slab, given by the reaction of the internal column, respectively.
The slabs were designed according to the requirements of NBR 6118: 2014. The dead load was composed by the structure self-weight and a load of 1 kN / m² due to the pavement. For the live load, three types of loading were considered: 2, 4 and 6.0 kN / m². The concrete cover was 2 cm.
Table 2 shows the slabs flexural reinforcement in the region of interest. Since this was a numerical analysis, non-commercial diameters were used for the reinforcing bars, so that all the slabs had the same relation between the calculated and the existing flexural reinforcement area.

2.1 Numerical model adopted to obtain the failure load of the slab

To obtain the failure load, a numerical model of finite elements was developed in ANSYS. The UPF (User Programmable Features) system, available in ANSYS, allowed the implementation of a constitutive model for concrete within the USERMAT subroutine. To describe the concrete behavior, two models were used, one for the concrete in compression and other for the tensile behavior. For the analysis of concrete behavior in compression, an elastoplastic model with linear hardening was adopted, composed by Ottosen

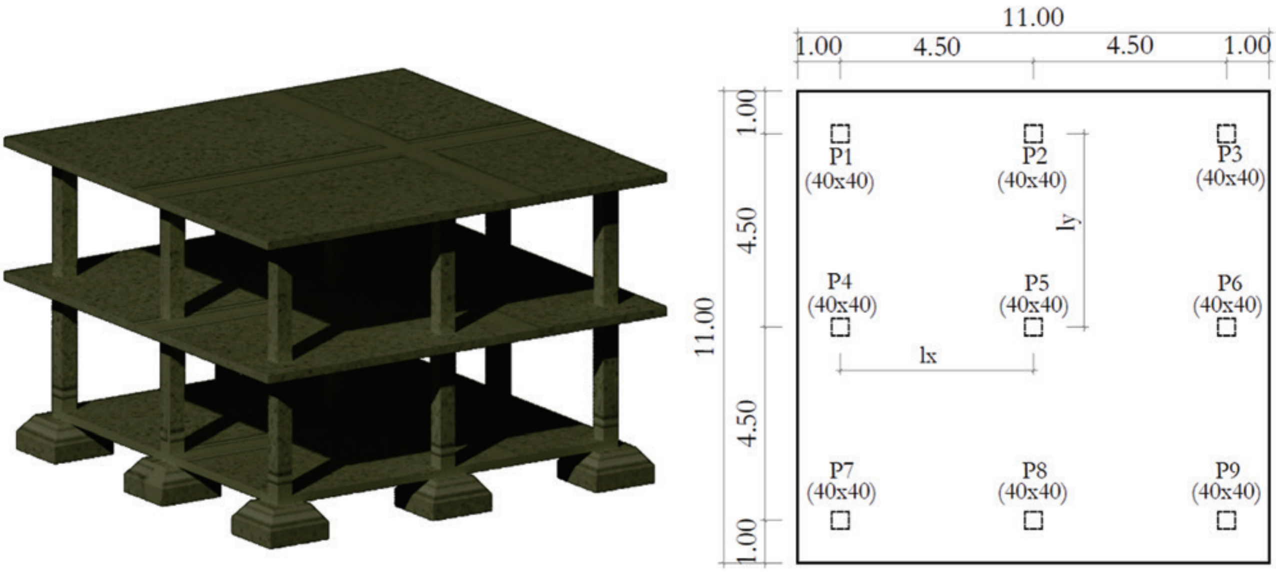

Figure 1
Floor plan of flat slab without shear reinforcement analysed in this study

Table 2
Flexural reinforcement in the region of interest

Slab	Top reinforcement (FEM) (in each direction)	Top reinforcement (EPA) (in each direction)	Bottom reinforcement (in each direction)	Progressive colapse reinforcement
L030216	21 ϕ 10.2 mm c. 10 cm	21 ϕ 7.7 mm c. 10 cm	12 ϕ 6.3 mm c. 17 cm	5 ϕ 13 mm
L030416	21 ϕ 11.1 mm c. 10 cm	21 ϕ 8.8 mm c. 10 cm	12 ϕ 6.3 mm c. 17 cm	4 ϕ 16 mm
L030616	21 ϕ 8.5 mm c. 10 cm	21 ϕ 9.8 mm c. 10 cm	12 ϕ 6.3 mm c. 17 cm	7 ϕ 14 mm
L030220	21 ϕ 9.6 mm c. 10 cm	21 ϕ 7.4 mm c. 10 cm	16 ϕ 6.3 mm c. 13 cm	5 ϕ 14 mm
L030420	21 ϕ 10.5 mm c. 10 cm	21 ϕ 8.3 mm c. 10 cm	16 ϕ 6.3 mm c. 13 cm	5 ϕ 16 mm
L030620	21 ϕ 8.3 mm c. 10 cm	21 ϕ 9.1 mm c. 10 cm	16 ϕ 6.3 mm c. 13 cm	5 ϕ 17 mm
L030224	21 ϕ 9.1 mm c. 10 cm	21 ϕ 7.2 mm c. 10 cm	19 ϕ 6.3 mm c. 11 cm	6 ϕ 14 mm
L030424	21 ϕ 9.9 mm c. 10 cm	21 ϕ 7.9 mm c. 10 cm	19 ϕ 6.3 mm c. 11 cm	5 ϕ 16 mm
L030624	21 ϕ 8.8 mm c. 10 cm	21 ϕ 8.6 mm c. 10 cm	19 ϕ 6.3 mm c. 11 cm	5 ϕ 18 mm
L060216	21 ϕ 10 mm c. 10 cm	21 ϕ 7.6 mm c. 10 cm	15 ϕ 6.3 mm c. 14 cm	5 ϕ 13 mm
L060416	21 ϕ 11.1 mm c. 10 cm	21 ϕ 8.7 mm c. 10 cm	15 ϕ 6.3 mm c. 14 cm	4 ϕ 16 mm
L060616	21 ϕ 8.4 mm c. 10 cm	21 ϕ 9.6 mm c. 10 cm	15 ϕ 6.3 mm c. 14 cm	5 ϕ 16 mm
L060220	21 ϕ 9.4 mm c. 10 cm	21 ϕ 7.4 mm c. 10 cm	21 ϕ 6.3 mm c. 10 cm	5 ϕ 13 mm
L060420	21 ϕ 10.3 mm c. 10 cm	21 ϕ 8.2 mm c. 10 cm	21 ϕ 6.3 mm c. 10 cm	5 ϕ 15 mm
L060620	21 ϕ 8.2 mm c. 10 cm	21 ϕ 9 mm c. 10 cm	21 ϕ 6.3 mm c. 10 cm	5 ϕ 17 mm
L060224	21 ϕ 9 mm c. 10 cm	21 ϕ 8.2 mm c. 10 cm	26 ϕ 6.3 mm c. 8 cm	5 ϕ 14 mm
L060424	21 ϕ 9.8 mm c. 10 cm	21 ϕ 8.2 mm c. 10 cm	26 ϕ 6.3 mm c. 8 cm	5 ϕ 16 mm
L060624	21 ϕ 8.8 mm c. 10 cm	21 ϕ 8.5 mm c. 10 cm	26 ϕ 6.3 mm c. 8 cm	5 ϕ 17 mm
L090216	21 ϕ 9.9 mm c. 10 cm	21 ϕ 7.6 mm c. 10 cm	19 ϕ 6.3 mm c. 11 cm	4 ϕ 13 mm
L090416	21 ϕ 11 mm c. 10 cm	21 ϕ 8.7 mm c. 10 cm	19 ϕ 6.3 mm c. 11 cm	5 ϕ 14 mm
L090616	21 ϕ 8.4 mm c. 10 cm	21 ϕ 9.6 mm c. 10 cm	19 ϕ 6.3 mm c. 11 cm	5 ϕ 16 mm
L090220	21 ϕ 9.4 mm c. 10 cm	21 ϕ 8 mm c. 10 cm	23 ϕ 6.3 mm c. 9 cm	4 ϕ 14 mm
L090420	21 ϕ 10.3 mm c. 10 cm	21 ϕ 8.1 mm c. 10 cm	23 ϕ 6.3 mm c. 9 cm	5 ϕ 15 mm
L090620	21 ϕ 8.8 mm c. 10 cm	21 ϕ 8.9 mm c. 10 cm	23 ϕ 6.3 mm c. 9 cm	5 ϕ 17 mm
L090224	21 ϕ 9 mm c. 10 cm	21 ϕ 8.8 mm c. 10 cm	29 ϕ 6.3 mm c. 7 cm	5 ϕ 13 mm
L090424	21 ϕ 9.8 mm c. 10 cm	21 ϕ 8.8 mm c. 10 cm	29 ϕ 6.3 mm c. 7 cm	5 ϕ 15 mm
L090624	21 ϕ 9.8 mm c. 10 cm	21 ϕ 8.8 mm c. 10 cm	29 ϕ 6.3 mm c. 7 cm	5 ϕ 17 mm

failure criterion [7], Von Mises plastification criterion and a hardening rule given by fib Model Code 2010 [6]. For the concrete tensile behavior, a linear elastic model was considered until the cracking of the concrete, and after cracking a model of smeared crack with a tension stiffening was used. This model is based on the formulation presented by Hinton [8] and Martineli's considerations [9]. For the steel reinforcement behavior, an elastoplastic model was adopted, represented by the internal model of ANSYS, denominated BISO (Bilinear Isotropic Hardening).

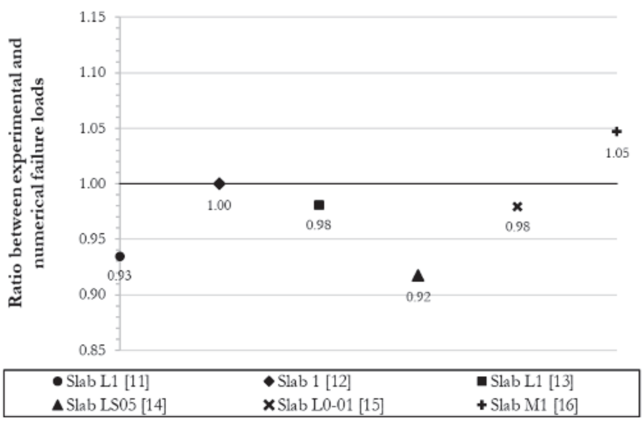

Figure 2
Ratio between experimental and numerical failure loads

The SOLID186 element was used to model the concrete in the slab, which is a quadratic, three-dimensional element with three degrees of freedom per node (translation according to the X, Y and Z axes), composed of 20 nodes. REINF264 element was used to model the reinforcement. This element is used in an embed way and presents only uniaxial stiffness. The nodal coordinates, degrees of freedom and connectivity of the element are the same as that of the concrete element. The detailed description of the model adopted to obtain the failure load in flat slabs can be found in the work of Silva [10].

The validation of the implemented model was done through comparison of numerical model results and experimental results found in the technical literature. Figure 2 shows these results. A good behavior of the implemented numerical model was verified through this analysis.

To reduce the computational time and to limit the rupture to the region in interest, the region of the internal slab-column intersection whose bending moment in the slab in a linear analysis is practically

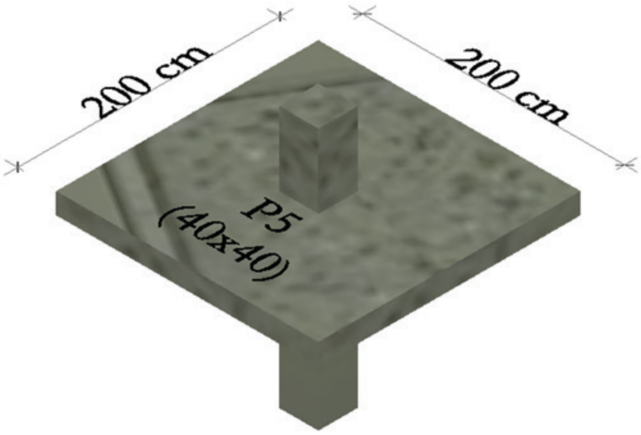

Figure 3
Slab region modeled to obtain the failure load

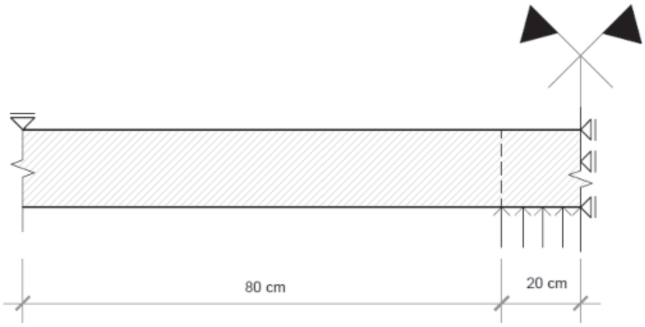

Figure 4
Boudary conditions of the studied slabs

null was studied, according to Figure 3, and only a quarter of this region was simulated due to symmetry.
It was considered as a boundary condition that this region is supported on the edges of the slab, not considering the membrane effect. In addition, in the regions of symmetry, it was considered that the normal displacements to the surface are equal to zero. These boundary conditions are shown in Figure 4.
The random variables adopted for this model were the concrete compressive strength and concrete tensile strength, concrete modulus of elasticity, yield strength of steel, slab thickness and distance from the center of the upper flexural reinforcement to the upper concrete face. The probability distribution, expected value, and coefficient of variation of these variables are shown in Table 3.

2.2 Numerical model adopted to obtain the internal column reaction

The internal column reaction was found through a finite element numerical model using the ANSYS software. The SHELL181 element was used for discretization of the slabs, and it is a linear element, of fourth nodes, with six degrees of freedom per node (translation and rotation according to the X, Y and Z axes). The BEAM188 element was used for discretization of the structure columns, and it is a two-node element, with six degrees of freedom per node (translation and rotation according to the X, Y and Z axes), and can be used with linear, quadratic or cubic interpolation functions. A linear elastic model was used to represent the behavior of the material. Thus, to obtain the reaction of the internal column, the whole structure shown in Figure 1 was modeled. The random variables adopted for this model are shown in Table 4.

Table 3
Statistical parameters of random variables of resistance function

Random variables	Type of probability distribution	Mean value	Coefficient of variation	Source of coeficiente of variation
Compressive strength of concrete	Normal	$f_{cm} = \frac{fck}{(1-1.645\,V_{fc})}$	0.15	[17]
Tensile strength of concrete	Normal	$f_{ctm} = 0.3(f_{ck})^{\frac{2}{3}}$, $f_{ck} \leq 50\,MPa$; $f_{ctm} = 2.12\ln\left(1+0.1(f_{cm})\right)$, $f_{ck} > 50\,MPa$	0.18	[4]
Modulus of elasticity of concrete	Normal	$E_c = 21500.\alpha_e.\left(\frac{f_{cm}}{10}\right)^{\frac{1}{3}}$	0.15	[4]
Yield strength of reinforcing steel	Normal	$\mu_{fy} = \frac{fyk}{(1-1.645\,V_{fy})}$	0.05	[17]
Slab thickness	Normal	$\mu h = h$	0.04	[18]
Distance between the geometric center of the upper flexural reinforcement and the concrete face	Normal	$\mu d' = d'$	0.125	[18]

Table 4
Statistical parameters of random variables of load function

Random variables	Type of probability distribution	Mean value	Coefficient of variation	Source of coeficiente of variation
Dead load	Normal	$\mu_g = 1.05g_k$	0.10	[17]
Live load	Gumbel	$\mu_q = \dfrac{q_k}{1 + 0.35\,V_q}$	0.25	[17]

2.3 Monte Carlo simulation and reliability index β

The Monte Carlo Method is available in ANSYS's Probabilistic Design System (PDS) tool, and the effect of the random variables on the results of the analysis was determined through this tool.
The simulations were performed until the stipulated statistical convergence criterion was achieved. This convergence consisted in verifying the mean and standard deviation for every 50 simulations, and an error of less than 0.5% of the mean and 1% in the standard deviation was necessary for the end of the simulations.
Due to a large number of simulations necessary to obtain the convergence of response parameters, a variance reduction technique was adopted to reduce the number of simulations. The adopted technique available at ANSYS was the Latin Hypercube sampling. In this technique, the range of possible values of each variable is divided into intervals, and a representative value is extracted from each range. These values are combined so that each representative value is considered only once in the simulation process.
The Kolmogorov-Smirnov statistical test was used to determine the probability distribution of the functions of R and S random variables. Using these distributions, the probability of failure was calculated by expression (5) and the reliability index of each of the studied slabs was calculated by expression (6).

The target reliability index adopted, based on Table 1, was 3.8. This index takes into account a high consequence of failure and relative cost of the safety measure and represents a probability of failure equal to 7.23. 10^{-5}. In addition, indexes between 3.1 and 3.8 were considered satisfactory in the reliability evaluation, although they are not ideal for this type of structure.

3. Results and discussion

The reliability index β for flat slabs without shear reinforcement ranged from a minimum of 2 for slab L030616 to a maximum of 5.56 for slab L090224, corresponding to the probability of failure of the order of 2.28 x 10-2 at 1.35 x 10-8. The index obtained for each slab is shown in Figure 5.
Analysis of figure 5 indicated that the reliability index β is smaller for the slabs whose internal forces and bending moments, for the slab design, were obtained by the EAP method. This can be explained by the fact that these slabs have lower flexural reinforcement ratios compared to the slabs that were designed with the bending moments obtained by the FEM. The increase in the flexural reinforcement ratio increases the contribution of the steel in the failure load value, and as the variability of the steel strength is small, the slabs gain resistance without increasing its variability, thus increasing the reliability index of these slabs.

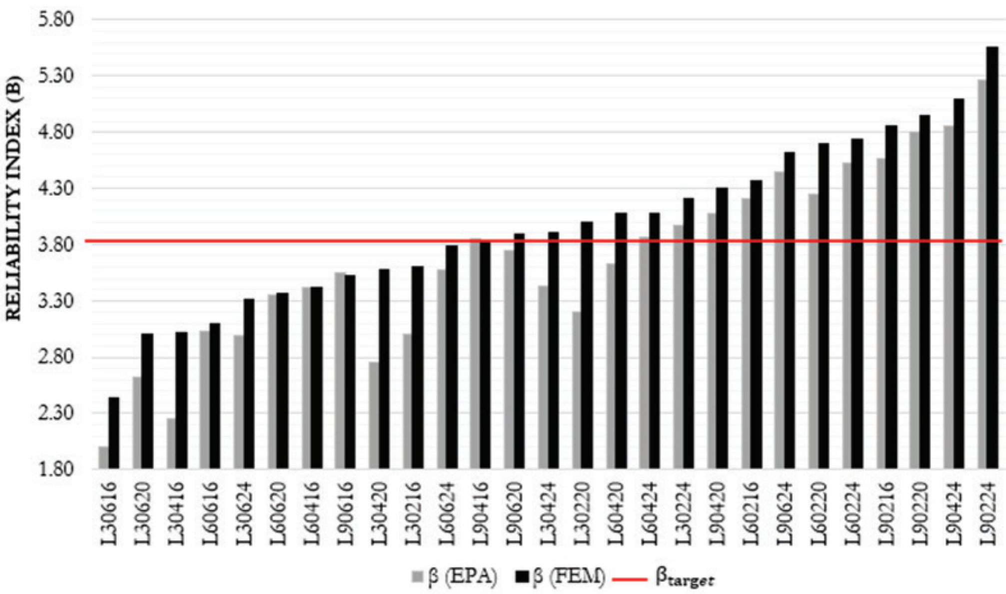

Figure 5
Reliability indices achieved by the flat slabs studied

The results demonstrated that the increasing of slab thickness contributes to the increase of the reliability index, as exemplified by Figures 6 and 7, and could be explained by the fact that the increase in thickness contributes to a gain in shear and flexural strength, inducing the increase in reliability index too. This fact can be explained by the significant increase in the average resistance of thicker slabs. Although the coefficient of variation of the resistance increases for these slabs, this increase is compensated by the significant rise of its mean value.

In Figures 8 and 9, it is evident that the live loading has a strong influence on the reliability index since with the increase in loading there is a decrease in the reliability index of all the slabs analyzed. This event can be explained by the increase in the coefficient of variation of the total load of the slabs subject to greater live loads. The concrete compressive strength also pointed an influence on the reliability indexes of the slabs in this study, as shown in Figures 10 and 11. Increasing the compressive strength of the concrete caused the failure probabilities of all the slabs to decrease and increased their reliability index.

4. Conclusions

In this study, the reliability of the internal column-slab connection of flat slabs without shear reinforcement was evaluated. The use of the finite element method for the analysis of slabs allowed the use of material models that accurately represented the physical reality of the problem, and allowed the use of a simulation method to evaluate the reliability of the structures. The Monte Carlo Method used for reliability analysis proved to be a practical and efficient method.

The reliability indexes obtained in this study were calculated based on Monte Carlo simulation data. Initially, a statistical study was performed on the data obtained in order to determine the probability distributions that best fit in the simulation data, and then the reliability index was calculated according to expression (5).

The analyzes obtained in this study pointed that, in general, the reliability indexes increased along with the increase of the flexural reinforcement ratio, decreased along with the increase of the live

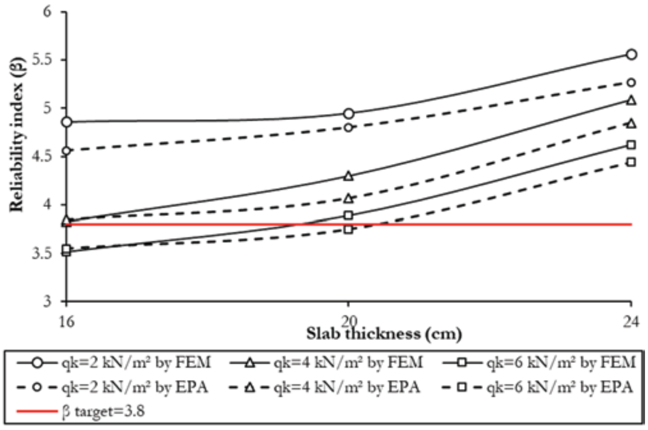

Figure 6
Analysis of the reliability index for slabs with f_{ck} = 90 MPa

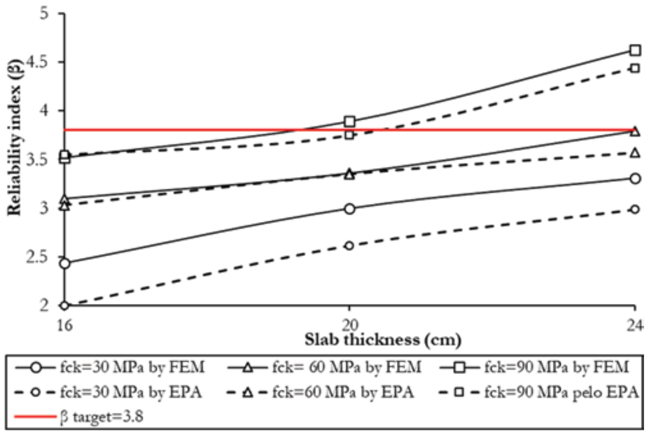

Figure 7
Analysis of the reliability index for slabs with live load of 6 kN/m²

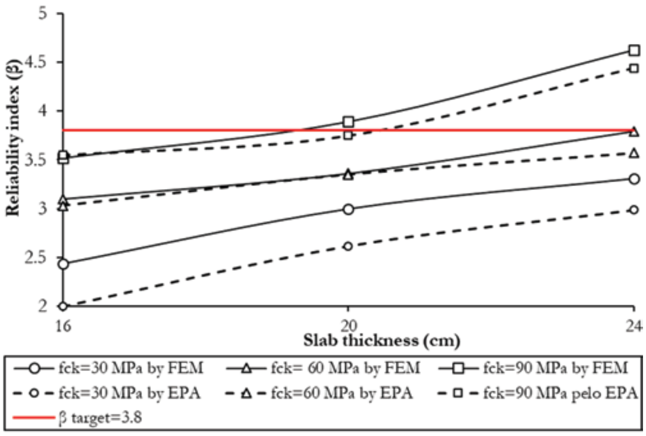

Figure 8
Analysis of the reliability index for slabs with f_{ck} = 30 MPa

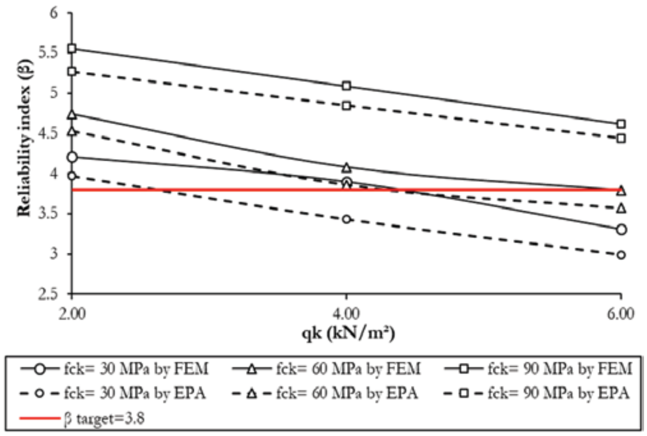

Figure 9
Analysis of the reliability index for slabs with thickness = 24 cm

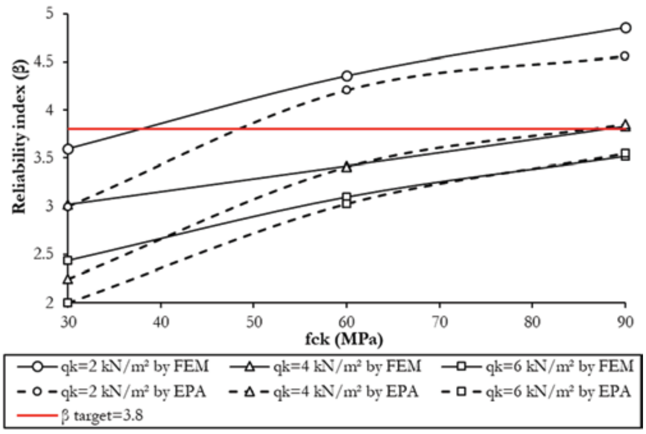

Figure 10
Analysis of the reliability index for slabs with thickness = 16 cm

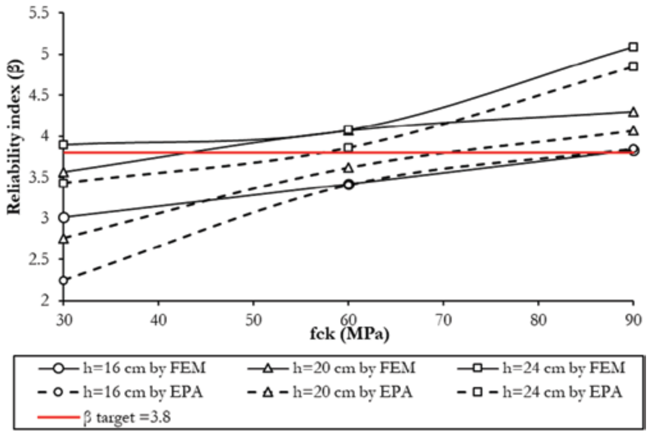

Figure 11
Analysis of the reliability index for slabs with live load of 4 kN/m²

load and grew along with the increase of the slab thickness and the compressive strength of the concrete.

Twenty-seven flat slabs had their internal forces and moments, for the design of the flexural reinforcement, calculated by the FEM, and only three slabs presented reliability indexes lower than the target reliability index considered satisfactory. However, from the 27 slabs analyzed by EAP method, seven had reliability indices below 3.1. In addition, from the ten slabs that obtained unsatisfactory results, only the slab L30216 presents a live load value that is commonly used in the civil construction.

Furthermore, it is important to point that in this study the relationship between the span and the effective depth of the slab was not considered, but this parameter has been shown to influence punching resistance in other studies, according to Muttoni [19]. Thus, it is evident the need for further studies to analyze the reliability of the NBR 6118: 2014 punch design method.

In future studies, more detailed experiments of flat slabs without shear reinforcement are proposed, considering parameters such as the relationship between the span and effective depth of the slab, the geometry of the column and the position of the column in relation to the edges of the slabs.

The results and conclusions established here are valid only for flat slabs with physical, geometric and load characteristics equal to the slabs considered in this study. For more comprehensive conclusions, more exhaustive probabilistic studies involving a greater number of parameters and random variables would be required.

Therefore, this study indicated that NBR 6118: 2014 is suitable for most flat slabs without shear reinforcement, and greater care is required when using the Approximate Elastic Process for the calculation of slab internal forces and bending moments and when using high live load values.

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