

Revised formula for predicting the long-term deflection multiplier of normal and high strength concrete

Fórmula revisada para predecir el multiplicador de deflexión a largo plazo de hormigón normal y de alta resistencia



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Abstract

High strength concrete (HSC) has found many applications in civil engineering structures such as in high-rise buildings, and bridges. The mechanical properties of HSC are sometimes different than of normal strength concrete (NSC). In particular, HSC possess lower creep strains compared to NSC. As a result, members constructed using HSC have been found to deflect less under sustained long-term loads. However, formulas used by current codes of practice such as ACI (318) code and Australian standard (AS-3600) for predicting the long-term deflections don't account for effects of HSC. This study aims to present a theoretical formula to calculate the long-term deflections for reinforced concrete beams made from NSC and HSC, taking into account the influence of HSC. The formula was derived from curve fitting analysis of long-term deflections obtained from several experimental tests available in literature. The presented equation considers the effects of several factors, such as compressive strength of concrete, and reinforcement at compressive zone, found in the experiments to have a significant impact on long-term deflections. The results of the equation were compared with experimental results of other researchers, and a good agreement was obtained. Following a parametric study, the long-term deflections were found to decrease to about 50% when increasing the concrete's compressive strength from 20 to 100 MPa. The compressive steel reinforcement was found less effective in the case of HSC.

Keywords: high strength concrete, creep, shrinkage, reinforced concrete, long-term, deflection.

Resumo

El hormigón de alta resistencia (HSC) ha encontrado muchas aplicaciones en estructuras de ingeniería civil, como en edificios de gran altura y puentes. Las propiedades mecánicas de HSC son a veces diferentes de hormigón de fuerza normal (NSC). En particular, HSC posee cepas de fluencia más bajas en comparación con la NSC. Como resultado, los miembros construidos con HSC se han encontrado para desviar menos bajo cargas sostenidas a largo plazo. Sin embargo, las fórmulas utilizadas por los códigos de práctica actuales, como el código ACI (318) y el estándar australiano (AS-3600) para predecir las desviaciones a largo plazo, no tienen en cuenta los efectos de HSC. Este estudio tiene como objetivo presentar una fórmula teórica para calcular las deflexiones a largo plazo para vigas de hormigón armado hechas de NSC y HSC, teniendo en cuenta la influencia de HSC. La fórmula se derivó del análisis de ajuste de curvas de las deflexiones a largo plazo obtenidos de varias pruebas experimentales disponibles en la literatura. La ecuación presentada considera los efectos de varios factores, como la resistencia a la compresión del hormigón, y el refuerzo en la zona de compresión, que se encuentra en los experimentos para tener un impacto significativo en las deflexiones a largo plazo. Los resultados de la ecuación se compararon con los resultados experimentales de otros investigadores, y se obtuvo un buen acuerdo. Después de un estudio paramétrico, se descubrió que las deflexiones a largo plazo disminuían a aproximadamente 50% cuando aumentaba la resistencia a la compresión del hormigón de 20 a 100 MPa. El refuerzo de acero compresivo se encontró menos eficaz en el caso de HSC.

Palavras-chave: hormigón de alta resistencia, fluencia, encogimiento, hormigón armado, a largo plazo, deflexión.

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1. Introduction

High-strength concrete (HSC) has been used in countless applications, especially for column and shear wall construction in high-rise buildings, and in bridges such as (prestressed girder bridges, box girder bridges, and cable-stayed bridges). An extensive list of field applications in which HSC was used in the construction of buildings, bridges, and other structures can be found in the guideline reported by the ACI committee 363R-10 [1]. The definition of high strength concrete has been under many changes over the time due to the constant addition of new concrete with ever increasing strengths. The 1997 edition of the ACI 363-97 report [2] used an f'_c of 41 MPa as the threshold for high strength; while in the current edition ACI 363R-10, an f'_c of 55 MPa is used. ACI 441R report on high-strength columns [3] used an f'_c of 70 MPa as the lower bound for HSC.

The mechanical properties, including compressive and tensile strengths, modulus of elasticity, Poisson's ratio, density, shrinkage, and creep, for HSC have been found to be different than corresponding properties of normal strength concrete (NSC) [1-11]. Given the difference in mechanical and other engineering properties between HSC and NSC, design equations, derived and established for NSC (e.g. flexural, shear, axial strengths, short- and long-term deflections), are being examined for extending their applicability for members constructed from HSC.

Using an HSC in the construction of a structure typically leads to an efficient and economic design by reducing the member's cross-section and dead loads, and increasing the member's span. However, the decrease in member's dimensions may create serviceability problems of excessive deflection due to the reduced stiffness and/or expected increase in slenderness ratio. It is, therefore, of paramount importance while optimizing the strength and weight requirements, to take appropriate measures to control deflections of HSC members under service loads. Serviceability requirement is more pronounced for HSC than NSC, given that HSC is typically used to provide a longer span and smaller cross-section (e.g. slender columns in high-rise buildings or long span girders in bridges) in which serviceability might control the design rather than strength [1-2].

Several studies have found that creep of HSC is generally lower than that of NSC [1-2, 12-14]. Similar studies have also found that HSC maintains larger long-term to short-term strength ratio than NSC [1-2, 15-16]. Some research has also found that HSC exhibits higher shrinkage than NSC made of similar materials.

1.1 Long-term deflection

Long-term deflection of structural members due to time-dependent variables of creep and shrinkage have been defined in multiple design codes using empirical equations that relate the long-term deflection with the elastic (short-term) deflection. In ACI 318-14 [18] code, the long-term deflection is determined by multiplying the short-term deflection by an empirical multiplier (λ), which considers the effects of compressive steel reinforcement (ρ') and duration of sustained loading, as following:

$$\lambda = \frac{\xi}{1 + 50\rho'} \quad (1)$$

Where: ξ is time-dependent factor for sustained loads that equals

1, 1.2, 1.4, and 2 for sustained load durations of three months, six months, one year, and five years or more, respectively. ρ' is the reinforcement ratio for non-prestressed compressive reinforcement. Although ACI code still uses equation (1) for both NSC and HSC, several experimental studies have found that (λ) multipliers for HSC are significantly lower than ACI proposed numbers [1, 4, 19-20]. This trend is expected, due to the fact that HSC possess lower creep coefficient than NSC as explained earlier. In addition, the ACI code equation doesn't include the effects of compressive strength (f'_c).

Similarly, researchers have found that the effects of compressive reinforcement in HSC members are lower than for NSC. In NSC members, compressive reinforcement helps in reducing creep of the concrete in the compressive region under sustained loads; while for HSC members, the concrete typically possesses low creep coefficient, hence diminishing the contribution of the compressive reinforcement for creep control [1].

Several modifications were proposed to revise the ACI multiplier (λ) and account for the effects of compressive strength and compressive reinforcement ratio. Luebkehan et al. [21] proposed the following equation for (λ):

$$\lambda = \frac{\mu_m \xi}{1 + 50 \mu_s \rho'} \quad (2)$$

where μ_m is a material modifier accounting for the effects of HSC on creep coefficient, and μ_s is a section modifier that takes into consideration the reduced influence of compressive reinforcement in controlling creep for HSC. Paulson et al. [4] revised equation (2) by combining μ_m and μ_s into one modifier. Issa et al. [20] tested five RC beams fabricated from HSC to evaluate the effects of concrete strength, reinforcement yield strength, span/depth ratio, and loading type (concentrated at mid-span, four-point, and uniform) on long-term deflections. The study reassured the influence of compressive strength; while the effects of compressive reinforcement were found to diminish upon increasing f'_c . Using plain longitudinal reinforcement led to an increase in long-term deflection. The study also presented a modification to the ACI (λ) multiplier. Numan et al. [22] conducted experimental tests on four simply supported, two-way slabs fabricated from NSC and HSC to examine the long-term deflection of both concretes. The study found that when increasing f'_c from 25 to 65 MPa (3626 to 9427 psi), the long-term deflection of the slabs reduced by 20%.

A similar methodology is followed by the Australian Standard (AS-3600-2009 [23]) to determine long-term deflections, by using a multiplier (kcs), which is a function of the compressive (A_s') and tensile (A_s) steel reinforcements, as seen in the following equation:

$$kcs = [2 - 1.2(\frac{A_s'}{A_s})] \geq 0.8 \quad (3)$$

Although the above equations are simple to use, their predictions of long-term deflections have been controversial [24-25]. This study aims at presenting a simple formula to accurately estimate the long-term deflection of flexural RC members made from HSC but also applicable for NSC. The proposed long-term deflection multiplier (λ) is derived from curve fitting of long-term deflections obtained from several experimental tests available in the literature. Two objectives were considered when developing the equation for

Table 1
Typical creep parameters (from [26])

Compressive strength, f_c'		Creep coefficient, ϵ_{ccu}
MPa	psi	
21	3045	3.1
28	4061	2.9
41	5946	2.4
55	7977	2.0
69	10007	1.6
83	12038	1.4

the multiplier: simplicity and inclusion of factors expected to influence the long-term deflection of HSC members.

2. Proposed equation for (λ) multiplier

Following the comparisons with experimental results (discussed in following section), the proposed model for the long-term deflection takes the following form:

$$\lambda_{prop} = 2.7\alpha \epsilon_{ccu} \frac{T^{0.3}}{10 + T^{0.3}} \tag{4}$$

$$\alpha = \frac{1}{1 + \left(\frac{16}{f_c'} \times \frac{\rho'}{\rho}\right)} \tag{5}$$

$$\epsilon_{ccu} = 4.14e^{-0.013 \times f_c'} \tag{6}$$

Where:

ϵ_{ccu} = ultimate creep coefficient, determined from curve fitting analysis of experimental data given by Nilson et al. [26] for different compressive strengths (f_c') in MPa, and presented in Table (1).

T = time of loading in months.

α = factor for the effects of compressive reinforcement.

λ_{prop} = proposed long-term deflection multiplier, dimensionless.

ρ' = compressive reinforcement ratio.

ρ = tensile reinforcement ratio.

3. Validation of proposed multiplier

The validity of the proposed multiplier is demonstrated in this section by comparisons with experimental results of RC beams obtained from the literature. It should be noted that the experimental data were selected to provide a wide range of values for factors such as beam sizes, span, compressive reinforcement, and strength of concrete. The experimental data were those reported in [4, 27-29].

3.1 Washa and Fluck tests

In the first set of experimental data, taken from [27], the authors measured deflections of 34 beams with different beam sizes, spans, and reinforcements, for a period of 2.5 years of sustained loading. All beams were simply supported, subjected to uniform loads. The dimensions and material properties of the tested beams are listed in Table. 2 along with the comparisons of the long-term deflection multiplier (λ) as obtained from the experiment; ACI equation (Eq. 1), AS-3600 formula (Eq. 3) and proposed equation (Eq. 4).

3.2 Corley and Sozen tests

In the second set of experimental data, given in [28], the study reported long-term deflections of simply supported beams under four-point bending loads, measured during 700 days of sustained loading. The dimensions and material properties of the tested beams are listed in Table (3) along with the comparisons of the long-term deflection multiplier (λ) as obtained from the experiment, ACI equation (Eq. 1), AS-3600 equation (Eq. 3), and the proposed equation (Eq. 4).

Table 2

Validation of proposed long-term deflection multiplier (λ) with experimental results by [27], time of sustained loading = 2.5 years

Beam No.	f_c' MPa	b mm	h mm	ρ %	ρ' %	L mm	Results of long-term multiplier, λ			
							Experimental	ACI, Eq. 1	AS Eq. 3	Proposed, Eq. 4
1	25.9	203	305	1.64	1.64	6096	0.75	0.94	0.8	1.07
2	25.9	203	305	1.64	0.76	6096	1.06	1.24	1.4	1.35
3	25.9	203	305	1.64	0	6096	1.63	1.72	2.0	1.73
4	20.8	152	203	1.66	1.66	6096	1.18	0.94	0.8	1.05
5	20.8	152	203	1.66	0.83	6096	1.61	1.21	1.4	1.34
6	20.8	152	203	1.66	0	6096	2.27	1.72	2.0	1.85
7	20.3	305	127	1.63	1.63	6340	1.00	0.95	0.8	1.04
8	20.3	305	127	1.63	0.81	6340	1.32	1.22	1.4	1.34
9	20.3	305	127	1.63	0	6340	1.94	1.72	2.0	1.86
10	20.1	305	127	1.54	1.54	3810	1.33	0.97	0.8	1.04
11	20.1	305	127	1.54	0.77	3810	1.38	1.24	1.4	1.34
12	22.2	305	127	1.54	0	3810	1.72	1.72	2.0	1.82
13	20.6	305	76	1.58	1.58	5334	1.09	0.96	0.8	1.05
14	20.6	305	76	1.58	0.79	5334	1.30	1.23	1.4	1.34
15	20.6	305	76	1.58	0	5334	1.93	1.72	2.0	1.86

Table 3

 Validation of proposed long-term deflection multiplier (λ) with experimental results by [28], time of sustained loading = 1.92 years

Beam No.	f'_c MPa	b mm	h mm	ρ %	ρ' %	L mm	Results of long-term multiplier, λ			
							Experimental	ACI, Eq. 1	AS Eq. 3	Proposed, Eq. 4
1	24	76	153	1.38	0	1830	1.47	1.61	2.0	1.67
2	24	76	110	2.06	0	1830	1.19	1.61	2.0	1.67
3	24	76	110	3.08	0	1830	1.54	1.61	2.0	1.67

3.3 Hajnal tests

Hajnal [29] conducted long-term tests on simply supported RC beams with varying span lengths subjected concentrated mid-span load. Deflections were measured during a period of (4.75) years of sustained loading. Table (4) lists the dimensions and material properties of the experimentally tested beams as well as the comparisons of the long-term deflection multiplier (λ) [from the experiment, proposed equation, and code predictions].

3.4 Paulson et al. tests

Within the fourth study, Paulson et al. [4] measured the long-term deflections of simply supported beams during one year of sustained loading. Testing variables were concrete's compressive strength (HSC vs. NSC) and compressive reinforcement ratio. Table (5) provides a list of beam's dimensions and material properties along with the results of the long-term deflection multiplier (λ), again from experimental results, proposed equation, and code predictions.

3.5 Results of experimental validation

Experimental data collected from literature discussed above, in which long-term deflection multiplier (λ) was measured, were used to validate the accuracy of the proposed multiplier and compare with two code predictions, ACI code (Eq. 1), and AS standard (Eq. 3). As can be seen in Table 2 to 5, the proposed equation (Eq. 4) provided a closer match with the experimental results than code equations, for most of the compared specimens. The average difference between predicted and experimental results of the (λ) multiplier for the proposed equation are: 0.54%, 20.8%, -12.6%, and 38.1% for test groups one to four, respectively, and 11.7% for all groups. For the ACI method, the average predicted/experimental differences are -7.4%, 16.4%, -15.2%, and 90% for test groups one to four, respectively, and 21% for all groups. While for the AS-3600 equation, the difference between predicted and experimental results are -2.9%, 44.7%, -14.7%, and 146.5% for test groups one to four, respectively, and 43.4% for all groups.

Table 4

 Validation of proposed long-term deflection multiplier (λ) with experimental results by [29], time of sustained loading = 4.75 years

Beam No.	f'_c MPa	b mm	h mm	ρ %	ρ' %	L mm	Results of long-term multiplier, λ			
							Experimental	ACI, Eq. 1	AS Eq. 3	Proposed, Eq. 4
1	24.5	130	191	0.72	0	6400	2.29	1.99	2.0	2.05
2	24.5	130	191	0.72	0	4800	2.58	1.99	2.0	2.05
3	24.5	130	191	0.72	0	3200	2.20	1.99	2.0	2.05

Table 5

 Validation of proposed long-term deflection multiplier (λ) with experimental results by [4], time of sustained loading = 1 years

Beam No.	f'_c MPa	b mm	h mm	ρ %	ρ' %	L mm	Results of long-term multiplier, λ			
							Experimental	ACI, Eq. 1	AS Eq. 3	Proposed, Eq. 4
1	90	127	254	1.50	0	5486	0.54	1.39	2.0	0.60
2	90	127	254	1.50	0.75	5486	0.53	1.01	1.4	0.55
3	90	127	254	1.50	1.50	5486	0.47	0.79	0.8	0.51
4	66	127	254	1.50	0	5486	0.70	1.39	2.0	0.83
5	66	127	254	1.50	0.75	5486	0.53	1.01	1.4	0.74
6	66	127	254	1.50	1.50	5486	0.53	0.79	0.8	0.66
7	37	127	254	1.50	0	5486	0.71	1.39	2.0	1.20
8	37	127	254	1.50	0.75	5486	0.56	1.01	1.4	0.99
9	37	127	254	1.50	1.50	5486	0.44	0.79	0.8	0.84

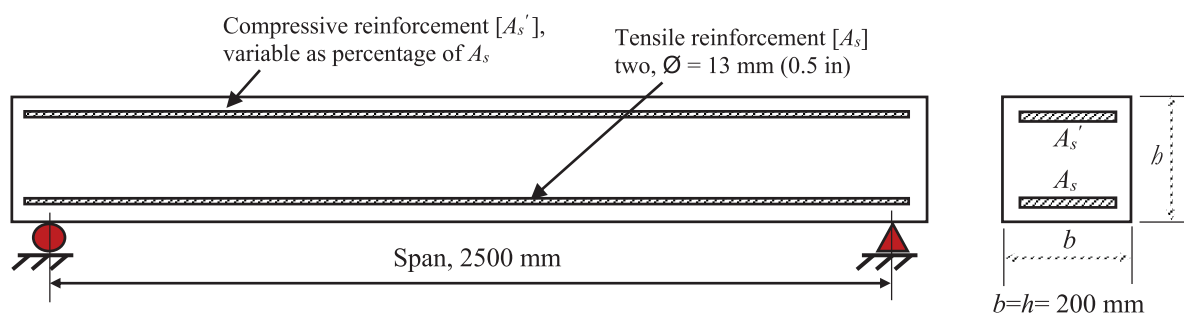


Figure 1
Dimensions of RC beam used in the parametric study

In the fourth test group, the proposed equation and code formulas were all overestimating long-term deflections. However, the proposed method presented the least overestimation, which was 52 and 108% more accurate than ACI and AS-3600 predictions, respectively. More importantly, and unlike the ACI and AS-3600 formulas, the proposed equation considers the effects of several factors (compressive strength, tensile and compressive steel reinforcement, cross-section dimensions, and span) found (from experimental investigations) to have a large impact on the long-term deflection of RC beams.

4. Parametric study

The proposed formula for long-term deflection multiplier (λ) is used to conduct a parametric study investigating the effects of compressive strength, and compressive/tensile reinforcement ratio. A typical simply supported RC beam, having a span of 2500 mm, and cross-section of 200 x 200 mm, is assumed for the study, as shown in Fig. 1. Tensile reinforcement, A_s , is assumed to consist of two longitudinal rebars with a diameter of 13 mm. Compressive reinforcement, A_s' , was varied as a ratio of the tensile reinforcement. The characteristic yield strength of steel reinforcement is taken as 413 MPa. Following sub-sections outline the factors investigated and results of the parametric study.

4.1 Effects of compressive Strength

Fig. 2 shows the relation between the time after sustained loading in months and (λ) multiplier for different compressive strengths (f_c'). It can be seen from the figure that increasing f_c' leads to a decrease in the long-term deflection multiplier (λ). λ was reduced by about 50% when f_c' was increased from 20 to 100 MPa. This can be attributed to the low creep coefficient for HSC as compared to that of NSC [1-2, 12-14]. In addition, ACI code formula (eq. 1) was also included in Fig. 2 for comparison purposes. As can be seen from the figure, ACI equation only provided a single relation between λ multiplier and time of sustained loading, while not considering the effects of f_c' . This in turn led to a reasonable prediction of λ multiplier for concrete strengths between 20 to 40 MPa, and an overestimation of λ multiplier for higher strengths, when compared with the proposed formula which takes into account the effects of f_c' .

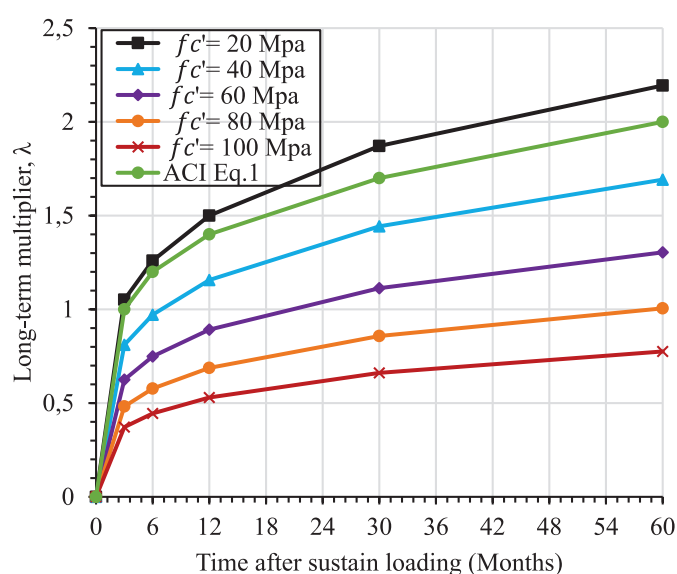
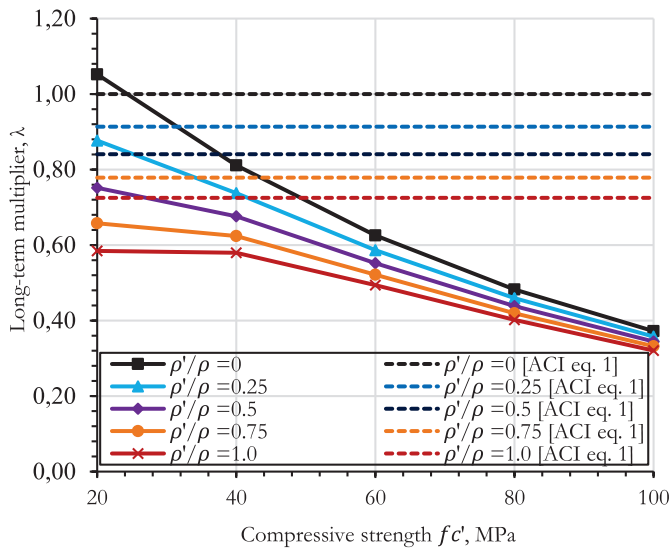


Figure 2
Effects of compressive strength (f_c') on the long-term deflection multiplier (λ)

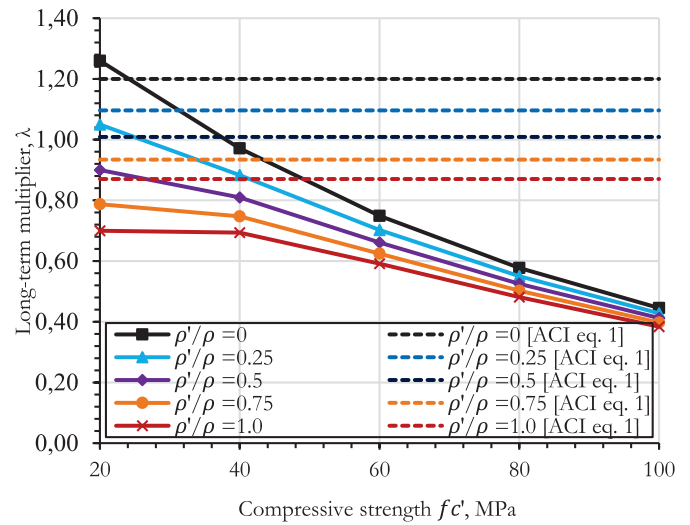
4.2 Effects of (compressive/tensile) reinforcement ratio

The effects of compressive reinforcement on the long-term deflection for the RC beam shown in Fig. 1 was investigated by varying the compressive reinforcement ratio (ρ') in reference to the tensile reinforcement ratio (ρ), which was kept constant and equals to the initial value in Fig. 1. Reinforcement ratios, ρ and ρ' , were determined by dividing the corresponding reinforcement (A_s and A_s') by the cross-section width (b) and effective depth (d). The compressive/tensile reinforcement ratios (ρ'/ρ) was varied from 0 to 1. Fig. (3) shows the relation between compressive strength (f_c') and (λ) multiplier for different reinforcement ratios (ρ'/ρ), at various times of sustained loading.

Fig. 3 shows that the long-term deflection [represented by (λ) multiplier] was reduced when the compressive reinforcement increased. However, Fig. 3 shows also that the effect of compressive reinforcement in reducing the long-term deflection depends on the compressive strength of concrete. When (f_c') is increased, the

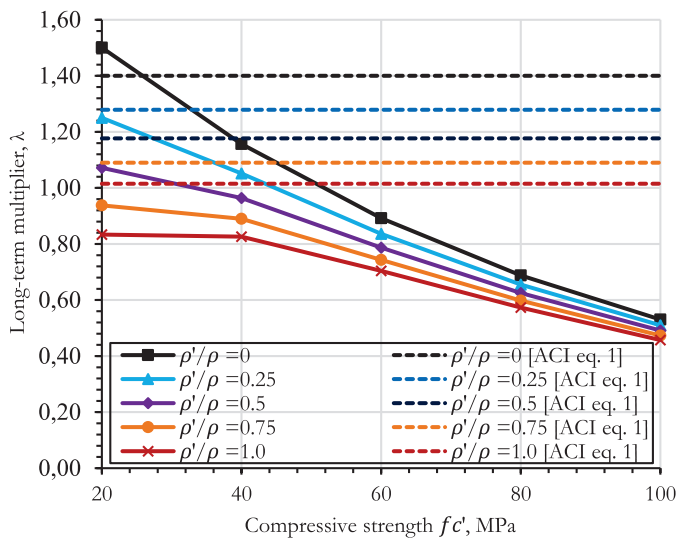


a After 3 months of sustained loading

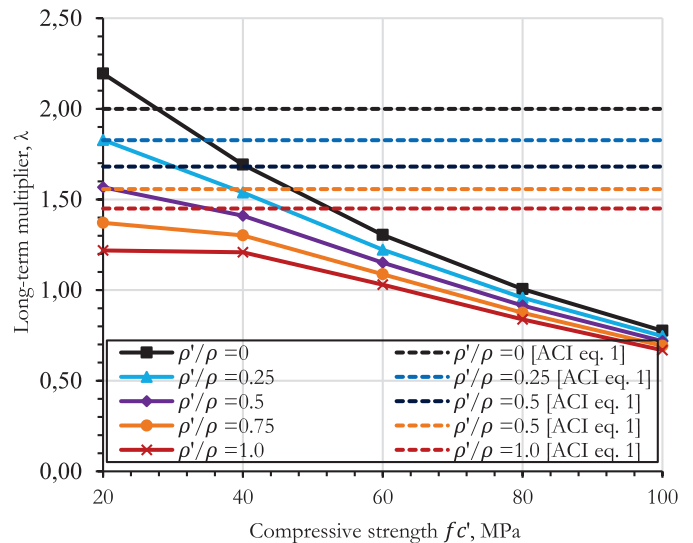


b After 6 months of sustained loading

Figure 3 Effects of the ratio between compressive to tensile reinforcement (ρ'/ρ) on the long-term deflection multiplier (λ)



c After 1 year of sustained loading



d After 5 years of sustained loading

Figure 3 Cont'd: Effects of the ratio between compressive to tensile reinforcement (ρ'/ρ) on the long-term deflection multiplier (λ)

effect of compressive reinforcement is reduced [Fig. 3 (a to d)]. As discussed in the introduction, the effects of compressive reinforcement diminish for HSC [1]. Shown in Fig. 3 also, the predictions of eq. 1 (ACI formula), where λ multiplier consistently reduces with increasing (ρ'/ρ) ratio. Due to absence of f'_c , the relation between λ multiplier and f'_c in eq. 1 is drawn in Fig. 3 as a constant line for each (ρ'/ρ) ratio. In addition, predictions of eq.1 and proposed formula are within comparable values for concrete strengths between

20 to 40 MPa, but eq.1 tends to overestimate λ multiplier for higher strengths, when compared with the proposed equation.

5. Conclusions

This study investigated the long-term deflections of RC members with focus on the effects of high strength concrete. The study presented an empirical equation to calculate the long-term deflection

multiplier (λ). The presented equation was derived from curve fitting analysis of several experimental tests and considered the effects of several factors such as compressive strength of concrete, and the ratio of compressive to tensile reinforcement ratio. A parametric study was performed to examine the effects of several parameters impacting the long-term deflection of RC beams. The following conclusions are drawn from the study:

- The proposed deflection multiplier presented a good match with experimental tests collected from the literature;
- The long-term deflection was found to decrease upon the increase of the compressive strength of concrete;
- The long-term deflection was found to decrease upon the increase of the compressive/tensile reinforcement ratio. However, the decrease in long-term deflection was less for HSC than for NSC.

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