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Determination of modulus of elasticity of concrete from the acoustic response

Determinação do módulo de elasticidade do concreto a partir da resposta acústica

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Abstract

Dynamic structural analysis is increasingly important for civil structures. In this context, non-destructive tests are a promising tool, as they allow obtaining integrated and comprehensive information about structure stiffness and damping and, moreover, may be repeated and compared over time. The determination of the modulus of elasticity of concrete by way of its acoustic response represents an innovative methodology to obtain a design parameter which associated with the compressive strength, achieves the guidelines for the design of structural elements in plain, reinforced and prestressed concrete. Test results show that the use of acoustic response-based tools in the determination of resonance frequency yields values with differences of less than 1% from those obtained by more commonly used non-destructive methods.

Keywords: dynamic experimental analysis; non-destructive testing; acoustic response; dynamic modulus of elasticity.

Resumo

A análise dinâmica de estruturas é cada vez mais relevante para estruturas civis. Nesse contexto, ensaios dinâmicos, de caráter não-destrutivo, são uma ferramenta promissora, pois permitem que sejam obtidas informações integradas e globais da estrutura a respeito da rigidez e do amortecimento e, além disso, podem ser repetidos e comparados ao longo do tempo. A resposta acústica é uma inovadora metodologia de ensaio não-destrutivo que visa a determinação do módulo de elasticidade do concreto, parâmetro que associado à resistência do concreto à compressão, configura-se como diretriz ao dimensionamento de elementos estruturais de concreto simples, armado ou protendido. Na comparação com metodologias usuais de ensaios não-destrutivos, os resultados demostraram que a utilização de ferramentas baseadas na resposta acústica apresentam diferença inferior a 1% na determinação da frequência de ressonância.

Palavras-chave: análise experimental dinâmica; ensaios não-destrutivos; resposta acústica; módulo de elasticidade dinâmico.

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1. Introduction

Dynamic non-destructive testing is a powerful tool for designers because it has the advantage of providing integrated and global information about the stiffness and the damping of the structure and, moreover, can be repeated and compared over time. This makes the information obtained from this type of test desirable in the preparation of structural designs, since the modulus of elasticity of the material, for example, can be monitored over the useful life of the structure from a single specimen, without the need for molding several specimens. It should be noted that from the modulus of elasticity it is possible to estimate the strength of the material, which further emphasizes the importance of such testing for designers.

In this context, this work aims to present a new method of non-destructive testing based on acoustic response and to compare it to methods of non-destructive testing already established, the end goal being their diffusion in the technical-scientific environment. The determination of the modulus of elasticity of concrete, a parameter associated with its compressive strength, provides guidelines for the dimensioning of structural elements of plain, reinforced or prestressed concrete.

A brief explanation is presented of the dynamic and experimental analysis of one of its potential applications: the characterization of materials -- more specifically, the determination of the dynamic modulus of elasticity of concrete. In what follows, the testing performed with cylindrical and prismatic specimens using the tools mentioned above is described and the results obtained are presented. Due consideration of the results obtained are presented at the end of the work.

2. Dynamic experimental analysis

2.1 Generalities

McConnell & Varoto [1], define vibration testing as the art and science of measuring and understanding the response of a structure when exposed to a specific dynamic environment. Testing should precisely simulate such environments in order to ensure satisfactory representation of the behavior of structures when exposed to similar vibration environments.

A series of applications of dynamic experimental analysis are described in several sources that deal with the subject. Ewins [2] and McConnell & Varoto [1] point to some applications of dynamic experimental analysis: (a) elaboration, verification, calibration, adjustment and correction of theoretical and numerical-computational models; (b) product development and qualification; (c) structural integrity and reliability verification; (d) production sampling; (e) monitoring operating conditions.

Among the many applications of dynamic experimental analysis, a combination of two of them stands out as the focus of this work: sampling production and structural integrity parameters, as concrete's modulus of elasticity is associated with both.

2.2 Dynamic testing to determine the modulus of elasticity

Dynamic experimental techniques can be classified into three categories: 1) impulse excitation; 2) sonic velocity (ultrasound) and 3) resonance frequency.

The resonance frequency technique, which by means of longitudinal vibrations, transverse and torsional, produces the fundamental resonance frequency of the structural element (prismatic or cylindrical), is one of the most widespread in Engineering. It is used for determining the dynamic modulus of elasticity, , the dynamic Poisson's ratio, the dynamic stiffness of the beam-column connection, and the damping of the material, among others. In the impulse excitation technique, in order to elicit an acoustic response, the specimen is supported by wires at its nodal points in the direction of vibration of interest and then receives a small stroke that induces an acoustic response. This response is composed of one or more natural frequencies of vibration, from which the modulus of elasticity of the material is calculated.

The sonic velocity technique is based on propagation time (flight time) of a sonic or ultrasonic pulse of short duration along the length of the specimen (frequencies above 20 kHz). This technique is regulated by the ASTM C597: 2009 [5]. Although widely used, a major uncertainty related to this measure comes from the importance of Poisson's ratio in these cases and of the impossibility of its calculation, since only the longitudinal velocity of sound is measured and it would be necessary to know the transversal velocity. Thus, the errors in the measure are proportional to the discrepancy between the true and estimated values of Poisson's ratio. Neville [4] considers this technique unreliable as some parameters, such as Poisson's ratio, can result in a reduction up to 11% in the value of the modulus of elasticity of.

To perform dynamic testing of this work, two acquisition systems were used. For both, the recommendations of ASTM - C215: 2008 [5] were complied with in full.

The first system, Sonelastic®, is a line of instrumentation solutions, developed by ATCP - Physical Engineering, for non-destructive determination of the modulus of elasticity and damping of materials derived from the natural frequencies of vibration obtained by the impulse excitation technique.

In this acquisition system, the modulus of elasticity and damping are calculated from the sound emitted by the specimen upon application of a small, mechanical impact. This sound or acoustic response is composed of the natural frequency of vibration of the specimen, which is proportional to the modulus of elasticity associated with the direction of vibration (Figure 1).

Figure 1 – Operation principle of the Sonelastic®

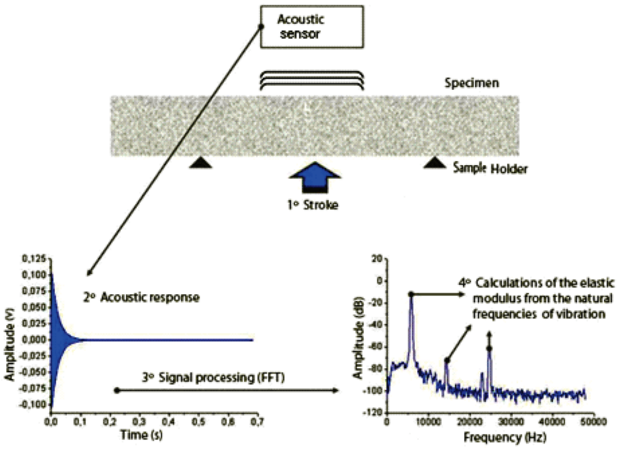
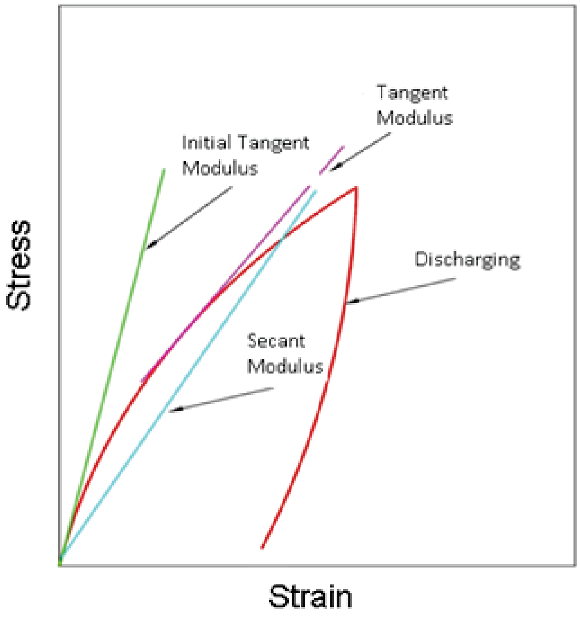


Figure 2 – Typical stress-strain curve of concretes



For simple geometries, such as a bar, cylinder, disc or plate, there is a univocal relation between the natural frequencies of vibration and the dimensions and mass of the specimen. Knowing the dimensions, mass and natural frequencies of vibration renders possible the calculation of elastic modulus. Damping is calculated from the logarithmic decrement of the amplitude of vibration from the decay rate of the signal [6].

One particularity of the equipment used in this paradigm for experimentation is the fact that the acoustic response of the specimen is captured using an acoustic sensor (microphone) instead of an accelerometer attached to the specimen. The latter occurs in most acquisition systems for non-destructive testing, such as the second acquisition system used in this work, the **ACE® System of Data Physics**. Because it does not require fixation of sensors, the use of an acoustic sensor makes the assembly of the testing apparatus even easier. Another important aspect of this system is that it does not require additional hardware; the audio cards already available in current computers can be used.

The **ACE®** System of **Data Physics** is a data acquisition system for accelerometry comprised of four channels, two for input and two for output. The system consists of a signal condenser plate coupled to a computer, a piezoelectric accelerometer (input) and an exciter (output), which can be an impact hammer or shaker. An advantage of this system consists in obtaining the Frequency Response Function (FRF), which relates system input and output. From the FRF, the modal parameters of the structure can be obtained. Each peak of amplitude is associated with a natural frequency (or resonant frequency) and a vibrational mode [7].

From an economic point of view, the acquisition systems similar to **ACE®** are more expensive than **Sonelastic®**, because in general the higher the number of channels, the more expensive the acquisition system.

3. Comparison between static and dynamic modulus of elasticity

From the typical stress-strain curve of a concrete body subjected to compression and tension stress, in the form of successive loading and unloading (Figure 2), it is observed that Young's term "elastic deformation modulus" can only, strictly speaking, be applied to the straight portion of the stress-strain curve, or when there is no straight portion at a tangent to the curve in the origin. In this case, it is called the initial tangent modulus of elasticity [8].

As the stress-strain curve of concrete presents nonlinear behavior, there is some difficulty in the accurate determination of a single value of static modulus of elasticity. Therefore, the use of dynamic experimental non-destructive methods that apply dynamic loads and do not interfere directly with the sample provides the value of the modulus of elasticity of the material more accurately. In addition, for the analysis of strains and stresses of dynamically loaded structures, it is more appropriate to use the dynamic modulus of elasticity ($E_{c,d}$) [8].

The $E_{c,d}$ can provide information about the deformability of concrete, the stiffness of a structural element, its relationship with other elements, the structural integrity, including those that are subjected to the usual static actions. The quality and reproducibility of the results of dynamic testing make the dynamic modulus a global parameter, obtained in an integrated manner and with a high degree of reliability. Being a property that can be obtained for the same specimen over a given period, the $E_{c,d}$ is used in testing that evaluates the changes of concrete subjected to chemical attack or freezing and thawing cycles. This leads to an additional attribute of dynamic non-destructive testing, the evaluation of some aspects of the durability of concrete.

Generally, when dynamic non-destructive testing is used to determine the $E_{c,d}$, it can be assumed that this value reflects the elastic behavior of concrete and that it is not affected by creep, since the vibration applied produces very low voltages. For this reason, the dynamic modulus of elasticity is approximately equal to the tangent modulus at the origin of the stress-strain curve determined in the static test, and is therefore larger than the static secant modulus. The $E_{c,d}$ is usually 20, 30 and 40 percent higher than the static modulus of deformation for concrete of high, medium and low resistances, respectively [8]. However, these authors did not indicate which static modulus of deformation this relationship is associated with, i.e. the tangent, secant or chordal. For some years now, researchers have sought to establish the relationship between static and dynamic moduli. This relationship is not easily determined by analysis of physical behavior, because the heterogeneity of concrete influences the two moduli in different ways [5]. Some empirical expressions that relate the static modulus ($E_{c,d}$) and dynamic modulus are presented below in Equations (1), (2), (3) and (4).

a) British Code BS 8110-2: 1985 [10]:

- For concrete with cement content less than 500 kg/m³ or concrete with aggregates of normal weight, the static modulus is given by Equation 1.

$$E_c = 1{,}25E_{c,d} - 19 \text{ (GPa)} \qquad (1)$$

- For concrete with cement content above 500 kg/m³ or concrete with lightweight aggregates, the static modulus is given by Equation 2.

$$E_c = 1{,}04E_{c,d} - 4{,}1 \ (\text{GPa}) \tag{2}$$

b) Lyndon & Baladran [11]:

$$E_c = 0{,}83E_{c,d} \tag{3}$$

c) Popovics [4]:

$$E_c = \kappa \cdot E_{c,d}{}^{1.4} - \rho^{-1} \tag{4}$$

Where, ρ is the density of concrete and is a constant that depends on the units of measurement.

However, it should be noted that these expressions apply to specific cases and there is no indication of the type of static modulus of elasticity to which these expressions refer.

Other researchers have tried to establish a correlation between the dynamic modulus of elasticity and the strength of concrete, but there is still no generalized relationship. There are, however, expressions that differ depending on the type of concrete used. Thus, the estimate of resistance based on the value of the dynamic modulus must be based on experimental results [4]. The relationship between the dynamic modulus ($E_{c,d}$) and compressive strength (f_{cub}), obtained in cubic specimen, established by the British Code CP 110:1972 [4] is given by Equation 5, being $E_{c,d}$ in GPa and f_{cub} in MPa.

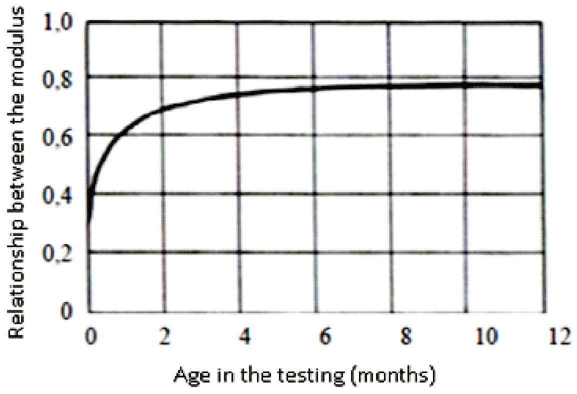
$$E_{c,d} = 7{,}6 \cdot f_{cub}{}^{0,33} + 14 \tag{5}$$

Some of the factors that influence the value of dynamic modulus of elasticity (and/or the relationship between this and the static modulus), or that may interfere in the obtaining of the resonance frequency used to calculate the $E_{c,d}$ are presented below:

a. Age - Neville [4] shows that for the same concrete the relationship between moduli (static and dynamic) approach a final value of 0.8 (Figure 3). In this case, samples with ages up to 12 months were analyzed. Han and Kim [12] found no major correlation between sample age and the relationship between the moduli, but all samples were tested up to 28 days of age;

b. Strength - The dynamic modulus increases with increasing strength and the higher the $E_{c,d}$, the smaller the difference between $E_{c,d}$ and E_c. This was due to the fact that the stretch elastic of stress-strain curve tends to be straighter as the fragility of the material increases [12]. Malhotra [9] cites the results of other authors who confirmed the increase of $E_{c,d}$ when the sample strength increased while maintaining the same conditions of curing. In different types of concrete, the relationship between moduli is greater as higher the strength of concrete becomes higher [4], as can be seen in Figure 4 and Figure 5;

Figure 3 – Relationship between static/dynamic modulus. (4)



c. Curing Temperature - With increasing temperature, the value of E_c approaches the value of $E_{c,d}$ [11];

d. Proportions of the Mixture and Aggregate Properties - The moduli of the constituent materials affect the deformability of concrete under the influence of dynamic actions. The dynamic modulus increases with a greater amount of aggregate with the same ratio of water/cement, while an increase in the amount of water in the mixture or the content of incorporated air reduces the dynamic modulus. As also happens with the static modulus of elasticity, the porosity affects the matrix and transition zone

Figure 4 – Relationship between static and dynamic modulus of concretes with different strengths. (4)

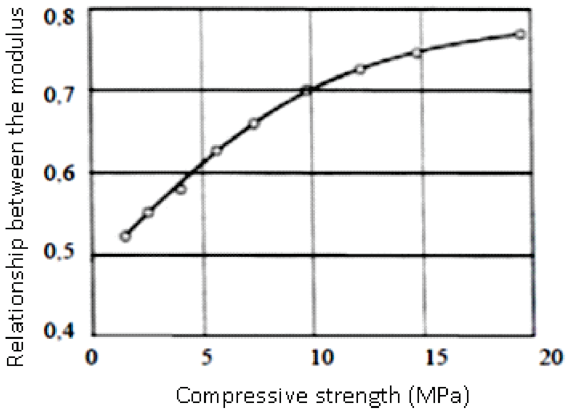
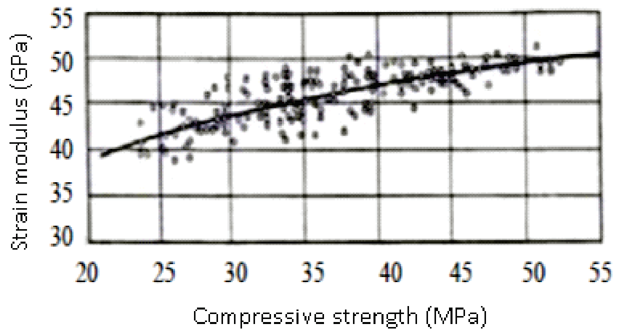


Figure 5 – Relationship between dynamic modulus and the respective compressive strength. (4)



of concrete, reducing its ability to restrict deformations $E_{c,d}$ [9];

- **e. Specimen Size -** Resonant frequency is inversely proportional to sample size.
- **f. Curing Conditions -** The change in modulus after three or four days of air drying is very small. When the concrete is kept moist, the modulus increases with age and if the concrete is exposed to drying, the modulus decreases with age. A general recommendation is to perform submerged curing and that the sample be saturated during testing to obtain satisfactory results [9].

Han & Kim [12] obtained experimental results about the effects of cement type, curing temperature, age and ratio of water to cement in relationships between the dynamic and static moduli of elasticity of concrete, and also between the dynamic modulus of elasticity and compressive strength. These authors noted that neither the type of cement nor age influence the relationship between $E_{c,d}$ and E_c , curing temperature being the only relevant factor.

The technique of sonic velocity was used by Han and Kim [12] to determine the dynamic modulus of elasticity in cylindrical specimens of 100 mm in diameter and 200 mm in height. When the initial chordal static modulus (defined by the authors as the declivity of the line drawn between the points of deformation equivalent to 10μ and 50μ) is used, it is possible to note that the relationship between E_c and $E_{c,d}$ is even closer. The authors concluded that there is a need for more experimental research to evaluate the influence of age on the relationship between the moduli, because the results obtained for samples in different age ranges varied, rendering comparison problematic.

Through experimentation, Han and Kim obtained Equation 6, which relates the static modulus to the dynamic modulus (in GPa):

$$E_c = E_{c,d} \cdot (1 - \alpha \cdot e^{-b \cdot E_{c,d}}) \qquad (6)$$

where α and b are constants obtained from experimental curve fitting and depend on the parameters studied (water/cement ratio, cement type and curing temperature), as well as the relationship between the dynamic modulus and compressive strength. The value of the dynamic modulus was found through relating the resonant frequency of the first longitudinal mode (f_1), the length of the cylinder (l) and the density of concrete (ρ_c), according to Equation 7:

$$E_{c,d} = 4 \cdot \rho_c \cdot \iota^2 \cdot f_1{}^2 \qquad (7)$$

Mesbah, Lachemi and Aïtcin [13] used the same technique, with the purpose of evaluating the evolution of resistance of high strength concrete for three mixtures, varying the water/cement ratio from 0.30 to 0.45. This allowed these authors to monitor the process from the beginning of the hardening of the concrete. The first measures in general were carried out only 10 hours from the time of molding. The results suggest that at low ages (under 24 hours), the dynamic modulus of elasticity is much higher than static modulus, and the ratio $E_{c,d}/E_c$ may reach a value of 4, depending on the water/cement ratio (w/c). The higher the w/c ratio, the higher the ratio $E_{c,d}/E_c$. However, the results showed that after 24 hours this ratio slightly decreases until finally stabilizing at 28 days. At this stage, the relationship $E_{c,d}/E_c$ seems to reflect little influence by the ratio w/c. Based on the results obtained for the type of aggregate and cement used, the authors proposed the following equation:

$$E_c = 9 \times 10^{-11} (65\, E_d + 1600)^{3.2} \qquad (8)$$

3.1 Poisson's ratios – static and dynamic

The Poisson's ratio (ν) is the ratio between the lateral and axial strains, in the elastic range, and represents another measure of the deformability of concrete. Usually, Poisson's ratio for compression loading is very similar to tension, but its value varies with the compressive strength of concrete, being lower for high strength concrete and higher for saturated and dynamically loaded concrete. However, that is not to say that Poisson's ratio varies with the age of concrete, the w/c ratio, the curing time, or the aggregate granulometry.

The expression for the calculation of Poisson's ratio uses the value of the modulus of elasticity (E) and transversal modulus of elasticity (or torsional) (G), and its value varies between 0.15 and 0.22 for concrete of normal weight. The dynamic determinations result in higher values, with an average of approximately 0.24 [4]. Knowing the $E_{c,d}$ (determined by the resonance frequency of torsional vibration) and $G_{c,d}$ (determined by the resonance frequency of torsional vibration), we then obtain the dynamic Poisson coefficient. The expression of the Poisson's ratio is (Equation 9):

$$\upsilon = \frac{E_{c,d}}{2 \cdot G_{c,d}} - 1 \qquad (9)$$

As of yet, there is no Brazilian standard for experimental deter-

Table 1 – Geometrical parameters and mass of the prismatic specimens

CP	L (mm)	D.P.*	b (mm)	D.P.*	h (mm)	D.P.*	m (kg)
01	748,67	0,58	151,17	0,76	152,00	1,00	42,50
02	750,67	0,58	152,33	0,58	154,00	1,73	43,10
03	750,17	0,76	152,67	0,58	152,33	0,58	42,50

* Standard deviations

mination of Poisson's ratio. The American Standard ASTM C-469: 2002 [14] establishes criteria for this testing. There is an indication of values to be used during the project, made by ABNT - NBR 6118: 2003 [15] (item 8.2.9) "for compression stresses lower than 0.5 f_c and tensile stresses lower than f_{ct}, the Poisson's ratio can be taken as equal to 0.2 and transverse modulus of elasticity equal to 0.40 $E_{c,s}$".

4. Materials and experimental program

4.1 Generalities and description of specimens

The experimental program of this work consists of two parts. The first corresponds to the characterization testing of concrete through impulse excitation technique using the **PC Based** Sonelastic® equipment, developed by ATCP - Physical Engineering. The second part consists of tests performed at the Laboratory of Structural Engineering at the São Carlos School of Engineering (EESC-LE). In this stage, the ACE® Data Physics acquisition system was used, which is the equipment usually employed in dynamic, non-destructive tests for material characterization at LE-EESC.

In order to validate the methodology and equipment used to determine the dynamic modulus of elasticity, testing was performed on cylinders and prisms of plain concrete, The same specimens were used in both stages.

However, beyond determining the dynamic modulus of elasticity, testing of axial compression with control of displacement and flexion at four points (for the prisms) was performed. This testing obtained the compressive strength, the tension in flexion and the static modulus of elasticity. For the static testing Vishay Instruments' System 5000 of was used.

Tables 1 and 2 summarize the data related to the geometry of the specimens and their respective masses.

4.2 Description of tests

4.2.1 Static testing

The determination of the static modulus of elasticity of concrete was done by the testing of axial compression, with displacement control, in cylindrical specimens (CP) of 150 mm in diameter and 300 mm in height, for 47 days, using the arithmetic mean of the values obtained for the E_c. The tests were performed in the universal testing machine, Instron, using two removable extensometers with bases of 200 mm, fixed to the specimen using rubber bands, in diametrically opposed positions. The acquisition system used was the System 5000 by Vishay Instruments.

The initial tangent static modulus of elasticity (E_c), was obtained according to ABNT - NBR 8522: 2008 [16], using methodology **A** described in Section 6.2.3.1 of this standard. The initial tangent modulus of elasticity is equivalent to the slope of the line secant to the stress-strain curve, passing between points at voltages equal to 0.5 MPa and 30% of rupture stress (in MPa).

To determine the tension strength in flexion (f_{ctM}) in prismatic specimens, the prescriptions of ABNT - NBR 12142: 1991 [17] were used. The CP used were the same prisms of 150 mm in width, 150 mm in height and 750 mm in length, used in the testing for determining the dynamic modulus of elasticity, because it is

Table 2 – Geometrical parameters and mass of the cylindrical specimens

CP	L (mm)	D.P.*	d (mm)	D.P.*	m (kg)
04	295,83	0,29	149,67	0,58	12,7
05	294,00	0,50	149,67	0,58	12,7
06	298,33	0,58	149,33	0,58	12,7
07	297,83	0,29	149,67	0,76	12,7
08	296,33	0,58	149,67	0,58	12,7
09	297,67	0,58	150,00	0,50	13,0

* Standard deviations

Figure 6 – Apparatus for dynamic testing of cylinder specimen used in the tests in ATCP

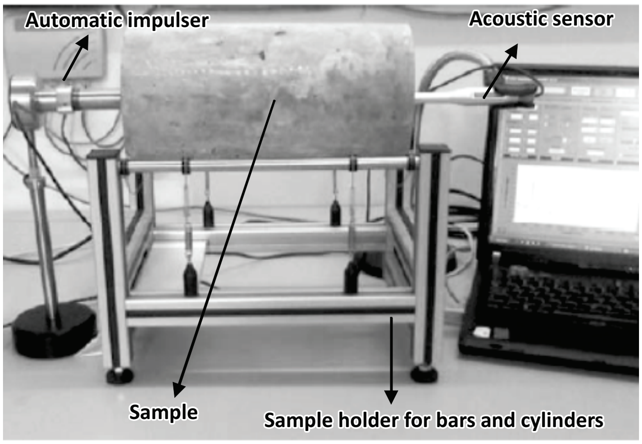
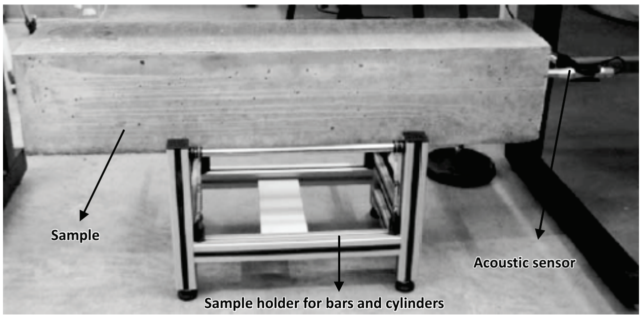


Figure 7 – Apparatus for the dynamic testing of prismatic specimen used in the tests in the ATCP



non-destructive testing. These were submitted to a static load of flexion (flexion test of four points) to the point of rupture. The flexion tests were performed in the Instron universal testing machine, using the same acquisition system, System 5000.

4.2.2 Dynamic testing

In order to determine the dynamic modulus of elasticity ($E_{c,d}$) obtained from the longitudinal resonant frequency, the prescriptions of the ASTM C215: 2008 [5] were adopted with regard to the positioning of the accelerometer, the point of application of excitation and the manner of simulation of the boundary conditions necessary to perform the test. Figures 6 and 7 show the layout of the tests of the CP performed at the ATCP - Physical Engineering. Figures 8 and 9 represent the tests performed in the LE-EESC.

5. Results analysis

Regarding testing of monotonic axial compression of the cylindrical CP, the average values of compressive strength and of static modulus of elasticity ($E_{c,}$), at 47 days, are presented in Table 3.

By comparison with the values obtained in the tests the calculation of the modulus of elasticity, was conducted according to ABNT - NBR 6118:2007 [15]. Item 8.2.8 of this standard shows two values of the modulus of elasticity, both calculated from the value of the compressive strength at 28 days.

The first value is known as the initial tangent modulus of elasticity ($E_{c,i}$), used in evaluating of the overall behavior of the structure and for the calculating of the loss of prestress. The standard also suggests a second value to be used in elastic project analysis, especially for the determination of force and for the verification of the serviceability limit states in structural elements undergoing tension or compression, which is known as the secant modulus of elasticity ($E_{c,s}$).

Figure 8 – Arrangement for dynamic testing of cylinder specimen used in the tests in the LE-EESC

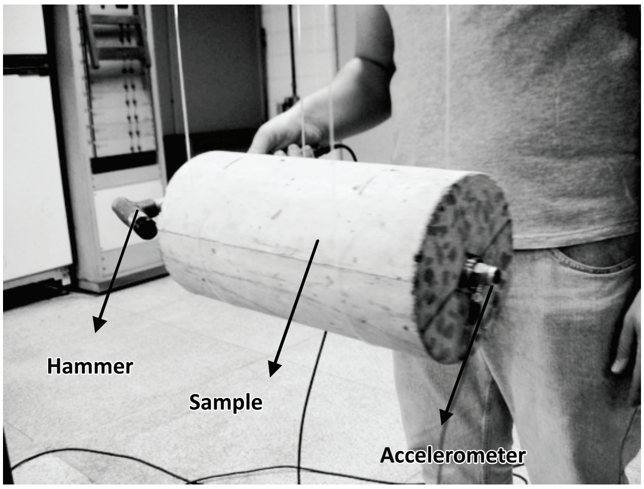


Figure 9 – Arrangement of dynamic testing for prismatic specimen used in the tests in the LE-EESC

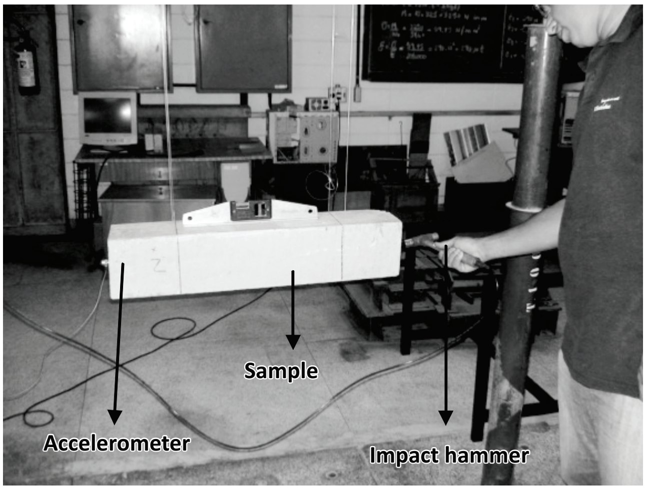


Table 3 – Results of static testing of cylindrical CP to 47 days – LE-EESC

CP-Cylindrical	$f_{c,47}$ (MPa)	E_c (GPa)
01	51,14	31,53
02	42,57	36,51
03	46,19	31,42
04	49,05	37,16
05	50,84	35,69
06	51,97	34,72
AVERAGE	48,63 ± 3,61	34,51 ± 2,48 (7,2%)

* Average ± standard deviations

The comparison of the values suggested for the project by the ABNT – NBR 6118:2007 [15] and obtained experimentally for the $E_{c,}$ from cylindrical specimens, is summarized in Table 5. The was 11.62% higher than the experimental E_c, while the $E_{c,s}$ was 3.97% lower than the experimental E_c, and this difference is considered small. Thus, for this research, since it the formulation proposed by ABNT - NBR 6118: 2007 [15] was considered in the estimation of the E_{ci}, the result obtained is contrary to standards of structural safety, as it overestimates the initial modulus of elasticity of concrete.

Regarding tension tests in flexion, it is typical that tension strength is expressed in terms of modulus of rupture. Mehta & Monteiro [4], confirm that the values for the modulus of rupture (F_{ctM}) obtained experimentally tend to overestimate the tension strength of concrete from 50 to 100%, mainly because the equation used in this determination assumes a linear stress-strain relationship in the concrete for the whole section of the beam. Moreover, in direct tension testing, the whole volume of the sample is subjected to applied tensions, while in the flexion testing only a small volume of concrete near the base of the sample is subjected to high tensions [8]. Considering the compressive strength (F_{c47}) obtained from cylindrical specimens (Table 3), the ratio (F_{ctM}/F_{c47}) should be situated at approximately 10% and 12% [8]. The ratio obtained in these trials was 9.52% on average, which demonstrates consistency with the values suggested in the literature.

The static elasticity modulus upon flexion ($E_{c,f}$) obtained with prisms at 48 days of age presented values up to 46.19% lower than those obtained with cylindrical specimens (Table 5). This difference is probably due to the use of the equations derived from the strength of materials for maximum elastic flexion which, according to Mehta & Monteiro [8], is an approximate measure. Thus, as concrete is a material that exhibits cracking in flexion, due to its heterogeneous and non-linear character, what can be expected is a lower value for the flexural modulus when compared to the compression modulus of elasticity.

Table 4 – Results of tension testing in flexion of the prismatic CP to 48 days – LE-EESC

CP – Prismatic	f_{ctm} (MPa)	$E_{c,f}$ (GPa)
07	4,26	19,960
08	4,38	20,819
09	4,35	22,255
AVERAGE	4,33 ± 0,062	21,01 ± 1,159

In dynamic testing the longitudinal dynamic modulus of elasticity was obtained. First, the natural frequencies of the prisms were determined, using the average value between the three tests. Tables 6 and 7 show the average values of the dynamic modulus of elasticity obtained for the cylinders and prisms based on the natural longitudinal frequency (the longitudinal direction corresponds to that parallel to the larger dimension of the specimen).

Comparing the results of the cylindrical and prismatic CP, the influence of the shape of the specimen on the value of dynamic modulus of elasticity can be perceived. The cylindrical CP showed lower values than those obtained with the prismatic CP. This observation can be identified in both the LE-EESC and ATCP-Physical Engineering tests.

As already mentioned, the $E_{c,d}$ for high-strength concrete (concrete that has strength above 40 MPa is considered high-strength [8]) is approximately 20% higher than the $\mathbf{E_c}$. The results obtained from the tests showed the same order of magnitude suggested by [8]; they were 20% higher than the E_c for cylindrical CP, while the prismatic CP showed values about 26% higher (Table 8). Considering the suggestions of BS 8110-2:1985 [10] as well as those of Lyndon & Baladran [11], for the calculation of the E_c from the experimental value of $E_{c,d,méd.}$ (for the cylinders, Table 6), a good approximation between the experimental value of E_c with those calculated analytically with the Equations 1 and 3 was observed (Table 9).

Finally the results for the damping coefficient (ξ_c) of the dynamic modulus of elasticity obtained with the resonant frequency of flexion ($E_{c,d,flexional}$), of the dynamic torsion modulus of elasticity

Table 5 – Initial tangent static modulus of elasticity – ABNT – NBR 6118:2007(15)

ABNT Standard – NBR 6118:2007 (15)	f_{ck28} = 48,63 MPa	$E_{c,i}$ = 39,05 GPa	$E_{c,s}$ = 33,19 GPa
Difference between the E_c		-11,62%	+3,97%
Difference between the $E_{c,f}$		-46,19%	-34,64%

Table 6 – Results of dynamic testing of cylindrical CP

CP - Cylindrical	$f_{n,long.}$ (kHz) LE-EESC	$E_{c,d}$ (GPa) LE-EESC	$f_{n,long.}$ (kHz) ATCP	$E_{c,d}$ (GPa) ATCP	ε_c ATCP
01	6,979	42,003	6,977	41,59	0,003327
02	6,925	41,076	6,996	41,56	0,003507
03	6,875	41,036	6,875	40,91	0,004751
04	6,981	42,169	6,932	41,33	0,002639
05	6,993	42,171	6,930	41,10	0,003048
06	6,925	41,911	6,922	41,98	0,003400
AVERAGE	6,946 ± 0,046	41,72 ± 0,53	6,938 ± 0,042	41,41 ± 0,38	0,003445
$E_{c,d,méd}$ = 41,565 GPa					

Table 7 – Results of dynamic testing of prismatic CP

CP - Prismatic	$f_{n,long.}$ (kHz) LE-EESC	$E_{c,d}$ (GPa) LE-EESC	$f_{n,long.}$ (kHz) ATCP	$E_{c,d}$ (GPa) ATCP	ε_c ATCP
07	2,814	44,281	2,804	43,56	0,003866
08	2,788	44,080	2,796	43,15	0,004254
09	2,794	43,654	2,796	43,12	0,003931
AVERAGE	2,798 ± 0,014	44,01 ± 0,320	2,798 ± 0,005	43,28 ± 0,246	0,004017
$E_{cdméd}$ = 43,645 GPa					

Table 8 – Comparative between $E_{c,d}$ and E_c

Type of CP	Relationship	Average
Cylindrical	$E_{c,d}/E_c$ (LE-EESC) = 1,199 $E_{c,d}/E_c$ (ATCP) = 1,209	1,204
Prismatic	$E_{c,d}/E_c$ (LE-EESC) = 1,254 $E_{c,d}/E_c$ (ATCP) = 1,275	1,265

Table 9 – Analytical calculations of E_c from the experimental $E_{c,d}$

Reference	$E_{c,d}$ exp. (GPa)	E_c exp. (GPa)	E_c calc. (GPa)	Diferença (%)
BS 8110-2: 1985 (10)	41,565	34,51	32,95	4,52
Lyndon & Baladran (11)	41,565	34,51	34,57	0,17

Table 10 – Other results of dynamic testing of prismatic CP

CP - Prismatic	$f_{n,flexional}$ (kHz)	$E_{c,d,flexional}$ (GPa)	$f_{n,torcional}$ (kHz)	$G_{c,d}$ (GPa)	$\upsilon_{c,d}$
07	1,031	43,45	1,648	17,85	0,22
08	1,021	41,99	1,644	17,69	0,19
09	1,032	42,50	1,645	17,59	0,21
AVERAGE	1,028 ± 6,084	42,65 ± 0,741	1,646 ± 0,002	17,71 ± 0,131	0,206 ± 0,015

($G_{c,d}$) and of the dynamic Poisson ($\upsilon_{c,d}$) are addressed. These parameters were determined only in the prismatic models (except the damping coefficient, which was also measured in cylindrical CP) with the Sonelastic® equipment.

The results presented in Table 10 indicate that the $E_{c,d,flexural}$ can replace the $E_{c,d}$ in dynamic testing of the prismatic CP without negatively impacting accuracy, since the difference between values obtained was of the order of only 2% of the $E_{c,d}$ [5]. This approach becomes interesting when we consider the difficulty of excitation in the longitudinal direction of the concrete prisms compared to excitation in the transverse direction.

The damping coefficient presented the behavior expected, i.e., it was very small. The damping of concrete elements presents a broad range of values, and depends greatly on the state of tension to which the element is submitted. At low voltage levels corresponding to the non-cracked state, the damping factor is less than 1% [7].

The dynamic torsional modulus of elasticity ($G_{c,d}$) presented about 40% of the $E_{c,d}$, indicating that the estimate of ABNT - NBR 6118:2007 [15] is quite reasonable, and even that this relationship is similar to that of static modulus of elasticity.

Using Equation 9, the values for the $\nu_{c,d}$ (Table 10) obtained, which were slightly higher than those recommended by ABNT - NBR 6118:2007 [15], although smaller than the value suggested by Neville [4].

6. Conclusions

Upon analysis of the results, the following can be consistently observed:

i. The value obtained for the secant static modulus, according to the ABNT - NBR 6118: 2007 [15] for the elastic analysis of the project, is slightly smaller (only 0.17%) than the average experimentally obtained value. However, for an global analysis of the structure, this result indicates the need for an experimental evaluation of the initial static modulus of elasticity, considering that the E_c was 11.62% less than the $E_{c,i}$;

ii. The flexural modulus of elasticity obtained from static tests was significantly lower than the E_c (Table 5). This is because of the attempt to describe a parameter of non-linear nature using a linear expression. Perhaps for this reason, the main objective of flexion testing was the determination of tension strength in flexion (or modulus of rupture) [8]. With regard to this parameter, the ratio f_{ctm}/f_{c47} of 9.52%, obtained in these trials, is considered acceptable, as the literature [8] suggests a ratio of 11 to 12%

iii. The determination of the modulus of elasticity of concrete from the non-destructive dynamic testing showed satisfactory and reliable results, since the $\mathbf{E_{c,d}}$ obtained remained in the range of 20 to 40%;

iv. It was possible to estimate satisfactorily the E_c from the $E_{c,d}$ using the formulations available in the literature (Table 9).

v. The shape of the sample influenced the experimental results. The values of dynamic modulus of elasticity obtained with cylindrical sample were 5% lower than the prismatic CP, on average;

vi. The different dates of the tests for the characterization of the $E_{c,d}$ did not influence the results significantly. It is supposed that the use of high early strength cement contributed to this result, since the tests were conducted 21 days after molding.

vii. Regarding the damping coefficient, the torsion dynamic modulus of elasticity and the dynamic Poisson coefficient obtained by **Sonelastic**®, even when not compared to the values obtained by the **ACE** system of **Data Physics**, are understood to be satisfactory. Because low levels of damping coefficient associated with the concrete were obtained, the relationship between $G_c/E_{c,d}$ resulted near 0.4, and the $\nu_{c,d}$ near 0.2 (Table 10).

viii. The **Sonelastic**® system was effective in determining the dynamic modulus of elasticity, having validated the results through comparison with those obtained by the consolidated acquisition **ACE**® system of **Data Physics**. It is noteworthy that the range of values obtained for the natural frequency of the specimens using **Sonelastic**® was less than 1%. Therefore, the objective of the work is considered to have been achieved.

Finally, it should be emphasized that dynamic non-destructive testing is a powerful tool for structural designers. The possibility of its application in the prefabrication of concrete structures, such as monitoring the development of concrete strength and modulus of elasticity, would be quite advantageous. This kind of testing is aligned with one of the premises of prefabrication -- the rationalization of construction, as less waste is generated, it is highly reproducible and allows the monitoring of the properties of concrete over the useful life of a single specimen.

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