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Contribution to the obtainment of concrete elastic modulus using micromechanics modeling

Contribuição à obtenção do módulo de elasticidade do concreto utilizando modelagem micromecânica



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Abstract

Mathematical expressions from national codes show that concrete elastic modulus is a function of concrete compressive strength. But concrete, regarded as a three-phase composite material, has elastic properties directly affected by the interfacial transition zone (ITZ), which is characterized by its higher porosity in comparison to the cement paste. Micromechanical models such as the Mori-Tanaka and three-phase sphere may be applied with good results when used to analyze the concrete. This paper presents a study on the evolution of the concrete elastic modulus, this study is carried out by the application of normative expressions and micromechanics models. Compared to the experimental results, a good fitting of micromechanical modeling with ITZ included is observed. Additionally, the quality of the NBR 6118:2003 and CEB-90 expressions is confirmed.

Keywords: concrete, elastic modulus, normative expressions, micromechanic, interfacial transition zone.

Resumo

Expressões presentes nas normas nacionais e internacionais relacionam o módulo de elasticidade do concreto com a resistência à compressão. O concreto, considerado como material compósito trifásico, tem suas propriedades elásticas diretamente influenciadas pela zona de transição (ITZ), a qual é caracterizada por sua maior porosidade em relação à pasta de cimento. Modelos de micromecânica como os de Mori-Tanaka e esfera de três fases podem ser aplicados com bons resultados quando utilizados para a análise do concreto. Este trabalho apresenta um estudo sobre o comportamento evolutivo do módulo de elasticidade do concreto, tal estudo é feito mediante a aplicação das expressões normativas e de modelos de micromecânica. Comparado-se com os valores experimentais produzidos percebe-se uma boa concordância quando a modelagem micromecânica leva em conta a ITZ. Também é confirmada a qualidade das expressões da NBR 6118:2003 e do CEB-90.

Palavras-chave: concreto, módulo de elasticidade, expressões normativas, micromecânica, zona de transição.

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1. Introduction

The mechanical properties of the concrete are fundamental in applications of such structural material [1]. Examples are the compressive strength and the elastic modulus, which are parameters directly related to the structural design.
In the 49th Brazilian Concrete Congress, carried out in 2007, controversial themes were debated, including "Elastic Modulus: Myths and Realities". Interesting aspects on this important property and the necessity of experimental evaluation were discussed. It was reported that, in general, engineers give to the elastic modulus a secondary role, using the mathematical expression recommended by the Brazilian code (NBR 6118:2003) [2].
According to this code, updated by NBR 6118:2007 code, the elastic modulus has to be determined by lab tests according to the NBR 8522:1984 code, update by NBR 8522:2008 [4], and, when lab tests are not conducted, a mathematical expression may be adopted, as a function of concrete compressive strength, yet an age of 28 days is assumed.
International codes such as Eurocode, ACI and CEB-FIP suggest different relationships for determining the concrete elastic modulus, which may be expressed as functions of compressive strength; it is also seen, in some mathematical expressions, that this parameter is related to the concrete density and to the type of aggregate used.
Papers such as [5-9] indicate that the concrete elastic modulus is dependent on the cement paste microstructure and on the type of aggregate used. It may be noted that:

- In [5], the influence of coarse aggregate type on concrete properties was studied, showing a relationship between concrete properties such as compressive strength, elastic modulus and the coarse aggregate source, particularly in the case of quartz.
- Analyses carried out by [6] intend to evaluate the probability that the failure surface occurs throughout the concrete coarse aggregate. In the analysis of fractured sections, the authors found that: when the water/cement ratio is increased, in the case of aggregates with characteristic diameter larger than 16 mm, the rupture probability throughout the aggregates decreases. It is important to understand that the characteristic diameter is more significant for high strength concrete and this probability depends not only on the strength, shape and size of aggregate, but is related as well to the reactivity of coarse aggregate with the cement paste.
- In [7], through the application of numerical modeling of constituents concrete phases, the influence of the interfacial layer and, consequently, the matrix of cement paste were observed. This interfacial layer reduces the contribution of the aggregate on the mechanical behavior of the concrete.
- In the paper [8], a numerical analysis is performed varying the volume fractions of concrete coarse and fine aggregates, observing higher values of the elastic modulus for 100% of coarse aggregate, of the total aggregate, and when the aggregates occupy 80% of the total mixture.
- The results presented in [9] emphasize, from the analysis of four types of aggregates, that for 0.44 water/cement ratio, aggregates with different mineralogical source do not have significant influence on the concrete elastic modulus or its compressive strength. But some differences were observed, from the mineralogical point of view, when the water/cement ratio was 0.26.

According to [10], the cement paste structure, in the aggregate particle neighborhood, is commonly very different from the structure of the bulk cement paste or mortar. Few aspects of the concrete behavior under stress can only be explained when the cement paste - aggregate interface is included as the third phase of the concrete microstructure.
This region, which has mechanical properties inferior to the rest of the cement paste, has influence on the mechanical behavior of the concrete under uniaxial stress. It is characterized by its higher porosity, in comparison to the cement paste.
Concrete can be dealt as a composite material. Linked to this consideration, there are models that deal with determining the properties of the composite material associated with the properties of their phases, these are called micromechanical models. These models are general ones, that are applied to composite materials of any nature and their application to concrete and mortar mechanics has proved to be satisfactory.
One of the basic problems in the composite material theory is the prediction of average or effective mechanical properties in terms of elastic properties and fraction volume of each phase [11].
In recent years, concrete has been assumed as a three-phase composite [12]. In this case, it is considered the presence of interfacial transition zone which, in many cases, was not taken into account. But it has been observed that its consideration has fundamental importance due to its influence on the composite mechanical behavior.
The proposed paper has as the objective of studying the behavior of the concrete elastic modulus over time, obtained by the application of normative expressions and micromechanics models.

1.1 Justification

Concrete elastic modulus is a necessary parameter in the structural analysis, particularly in the deformation and displacement analyses. The mathematical expressions proposed for the direct determination of the concrete elastic modulus in various codes and scientific articles do not take into account the influence of the concrete phases in the composite mechanical behavior.
It is presented by [13], through the verification of the Hashin-Strikman bounds, that when the concrete is considered as a composite material, this should not be dealt as a two phases material, it is suggested the incorporation of the interfacial transition zone as the third phase of the composite material.
The quantification of the mechanical properties of interfacial transition zone is still a problem due to the complexity of its behavior and there is no consensus on its exact thickness. The knowledge of these properties is a very important issue for the mechanical characterization of concrete or mortar, since the behavior of these composite materials strongly depends on the properties displayed by the interfacial transition zone.
The studies presented in the technical literature on the application of micromechanical models for the analysis of concrete properties were created for specific concrete ages and there isn´t any study on its application to predict the concrete behavior over time. For example, to estimate the elastic modulus according to NBR 6118:2003 code, it is necessary to know the concrete compressive strength in the respective age.

2. Aggregates and Interfacial Transition Zone

The aggregates are essential components in the concrete mix and

they occupy the largest volume among the mixture phases. The quality of the aggregates has direct influence on the concrete properties and their defects may cause unsatisfactory performance because they have direct influence on the behavior of the transition zone, although their compressive strength, in the majority of the cases, is not responsible for the rupture of the concrete.
The increase of the maximum size of the aggregate to a specified limit or using densely graduated aggregate leads to the growth of the concrete elastic modulus [14].
The facts that micro cracking is initiated in the interface between the coarse aggregate and cement paste and that, at the rupture point, the cracks include this interface demonstrate the great importance of this concrete phase [15].
The properties of the interfacial transition zone, specially the void volume and the micro cracks, have great influence on the concrete elastic modulus and its rigidity.
In the composite material, the transition zone serves as a link between two constituents, the mortar matrix and the particles of the coarse aggregate. Then, especially in the cases where the individual constituents have high rigidity, the rigidity of the composite material could be lower because of the voids and micro cracks present in the transition zone, which do not allow energy transference [10].

3. Normative Expressions

3.1 NBR 6118:2003

According to NBR 6118:2003, the concrete elastic modulus should be obtained using the test described by NBR 8522:1984 (update to NBR 8522:2008), where it is considered the initial tangent elastic modulus at 30% of ultimate compressive strength of concrete, previously obtained testing a sample.
To estimate the initial tangent elastic modulus, NBR6118:2003 code recommends that, without experimental data of a given sample, after 28 days of age, it may be used the Equation (1).

$$E_{ci} = 5600 f_{ck}^{1/2} \qquad (1)$$

where E_{ci} is the initial tangent elastic modulus and f_{ck} is the characteristic compressive strength of concrete, both in MPa.
According to the same code, equation (1) may be used to evaluate the elastic modulus of the concrete at any age greater than or equal to 7 days, simply replacement f_{ck} by f_{ckj} given for the required age.
According to [16], reviewing NBR 6118:2003, updated in 2007, it is recognized that the elastic modulus is linked to the average value and not to the characteristic value, justifying the choice of the latter by the unknowing of the average compressive strength in the design stage.
Again according to [16], in this review was suggested an expression that took into account the type of the aggregate and concrete consistency, as shown in Equation (4):

$$E_{ci} = a_1 a_2\, 5600 \sqrt{f_{ck}} \qquad (4)$$

Table 1 – Coefficient a_1

Coarse Aggregate	a_1
Basalt, dense sedimentary limestone, diabase	1.1 to 1.2
Granite and gneiss	1.0
Metamorphic limestone, metasediments	0.9
Sandstone	0.7

Table 2 – Coefficient a_2

Consistency	a_2
Fluid	0.9
Plastic	1.0
Dry	1.1

where a_1 is a parameter related to the type of aggregate and a_2 is related to the concrete consistency. In [18] are pointed out values that would be representative for these coefficients. See Tables 1 and 2.
In order to obtain the expression used by the NBR 6118:2003, there were carried out a series of tests using materials from some specific regions of Brazil, so the results may be strongly influenced by the material source, particularly in the case of the coarse aggregate.

3.2 CEB

To determine the elastic modulus, CEB-90 adopted Equation (5), valid for concrete compressive strength up to 80 MPa [19-21]:

$$E_{ci} = 21500 \cdot \sqrt[3]{\frac{f_{ck}+8}{10}} \qquad (5)$$

where E_{ci} is the initial tangent elastic modulus at the age of 28 days and f_{ck} is the characteristic compressive strength of the concrete, both in MPa. The term ($f_{ck}+8$) represents the average compressive strength of concrete (f_{cm}).
For different ages, CEB adopts an expression as a function of the compressive strength of the concrete at that age (f_{cj}), as shown in Equation (6):

$$E_{ci} = 21500 \cdot \sqrt[3]{\frac{f_{cj}}{10}} \qquad (6)$$

where f_{cj} is the average compressive strength of concrete, in MPa, at the desired age.
Equation (6) is applied to concrete made with aggregates of quartz (granite and gneiss). For aggregates of basaltic source, the value of the elastic modulus must be multiplied by 1.2. For aggregates from limestone and sandstone, the multiplicative factors should be 0.9 and 0.7, respectively [16].
The secant elastic modulus may be calculated by the same way shown in NBR 6118:2003, through its relationship with the initial tangent elastic modulus.

3.2 EUROCODE 2

The expression proposed by Eurocode 2 (1992) for determining the initial tangent elastic modulus, without experimental values and in situations where precision is not required, may be determined by Equation (7) [22]:

$$E_{ci} = 9500 \cdot \sqrt[3]{f_{ck} + 8} \quad (7)$$

where f_{ck} is given in MPa. This equation is valid for concrete made with coarse aggregate of quartz [21].
ACI (American Concrete Institute)
In [20] is shown the relationship for determining the elastic modulus according to ACI 363 (1997), where the determination of the secant elastic modulus of the concrete to a stress level of 45% of rupture compressive strength is presented in Equation (8):

$$E_{cs} = \rho^{1,5}\left[3320\sqrt{f_{cj}} + 6900\right] \quad (8)$$

where r is the density of concrete, in kg/m³, and f_{cj} is the compressive strength of the concrete at the specific age, given by MPa.
The equation proposed by ACI 363 expresses the concrete elastic modulus in relation to its density, assuming this correlation due to the fact that the denser a solid body, the greater its rigidity and its strength to deformation [20].
According to [16], ACI 318 (1995) adopted for the determination of the secant elastic modulus the Equation (9):

$$E_{cs} = 4730\sqrt{f_{ck}} \quad (9)$$

4. Micromechanical Models

In table 3 are shown the variables used to understand the models presented below.

4.1 Mori-Tanaka Model

The Mori-Tanaka model is evaluated for two phase composite materials, where the RVE (representative volume element) and the inclusion have coaxial ellipsoidal shape [23-25].
It is assumed that, inside the inclusion, Eshelby formula may be applied ($<\varepsilon>_I = \varepsilon^0 + S : \varepsilon^*$), where ε^* is the inclusion eigenstrain, according to the equivalent inclusion method given by Equation (10):

Table 3 – Notations of the micromechanical models variables

Variable	Description
ε^0	Uniform strain
ε^*	Inclusion eigenstrain
S	Eshelby tensor
C	Stiffness tensor
$<\varepsilon>$	Average strain
$\bar{C}$	Effective stiffness tensor
f_I	Volume fraction of inclusion
Ω_0	Ellipsoidal domain of Mori-Tanaka model
A	Strain concentration tensor
$<\sigma>$	Average stress
K	Bulk modulus
K_{inf}	Inferior bound of Hashin-Shtrikman for K
K_{sup}	Superior bound of Hashin-Shtrikman for K
G	Shear modulus
G_{inf}	Inferior bound of Hashin-Shtrikman for G
G_{sup}	Superior bound of Hashin-Shtrikman for G

$$\varepsilon^* = \left[(C_M - C_I)^{-1} : C_M - S\right]^{-1} : \varepsilon^0 \quad (10)$$

where S is the Eshelby tensor, ε^0 is a uniform strain applied on the bound, C_M and C_I are stiffness tensors of matrix and inclusion, respectively.
If ε^* is assumed as constant, the stress and strain also are constants and their values coincide with the average.
The average strain in the inclusion may be written as Equation (11):

$$<\varepsilon>_i = \{I + S : \left[(C_M - C_I)^{-1} : C_M - S\right]^{-1}\} : \varepsilon^0 \quad (11)$$

where I is the unit tensor.
The premise of Mori-Tanaka model is showed by Equation (12):

$$<\varepsilon>_M = \frac{1}{V_M}\int_{V_M}(\varepsilon^0 + \varepsilon^d(x))dV_M = \varepsilon^0 + \frac{1}{V_M}\int_{V_M}\varepsilon^d(x)dV_M = \varepsilon^0 + <\varepsilon^d>_M \quad (12)$$

where $<\varepsilon>_M$ is the average strain tensor and $<\varepsilon^d>_M$ is the eigenstrain tensor.
According to this method the term $<\varepsilon^d>_M$ approaches to zero, which is valid for an ellipsoidal volume [26], and the average strain in the composite may be written through the average strain and volume fraction of each constituents ($<\varepsilon>= f_I <\varepsilon>_I +(1-f_I)<\varepsilon>_M$). Applying the Eshelby equation and using $<\varepsilon>_M=\varepsilon^0$, Equation (13) is found:

$$<\varepsilon>=\{I+f_I S:\left[(C_M-C_I)^{-1}:C_M-S\right]^{-1}\}:\varepsilon^0 \quad (13)$$

where $<\varepsilon>$ is the average strain tensor in the composite and f_I is volume fraction of inclusion.
The composite constitutive equation, based on the average stress and the average strain tensors, leads to the effective elastic tensor determined by Equation (14):

$$\bar{C}=C_M:\{I+f_I(S-I):\left[(C_M-C_I)^{-1}:C_M-S\right]^{-1}\}:\{I+f_I S:\left[(C_M-C_I)^{-1}:C_M-S\right]^{-1}\} \quad (14)$$

In the inclusion domain Ω_0, the average strain is written by Equation (15):

$$<\varepsilon>^{\Omega_0}=<\varepsilon>_M+S^{\Omega_0}:\varepsilon^* \quad (15)$$

where $<\varepsilon>^{\Omega_0}$ is the average strain in inclusion domain.
By applying the equivalent inclusion method in domain Ω_0, it may be written Equation (16):

$$C^{\Omega_0}:<\varepsilon>^{\Omega_0}=C_M:(<\varepsilon>^{\Omega_0}-\varepsilon^*) \quad (16)$$

where C^{Ω_0} is the elastic tensor for the inclusion domain.
Writing $<\varepsilon>^{\Omega_0}$ as function of ε^*, Equation (17) is obtained:

$$<\varepsilon>^{\Omega_0}=A^{\Omega_0}:\varepsilon^* \quad (17)$$

where A^{Ω_0} is given by (Equation 18):

$$A^{\Omega_0}=(C_M-C_I)^{-1}:C_M \quad (18)$$

Combining Equations (15) and (17), results Equation (19):

$$\varepsilon^*=(A^{\Omega_0}-S^{\Omega_0})^{-1}:<\varepsilon>_M \quad (19)$$

Substituting ε^* in Equation (17), Equation (20) is encountered:

$$<\varepsilon>^{\Omega_0}=A_{dil}^{\Omega_0}:<\varepsilon>_M \quad (20)$$

where $A_{dil}^{\Omega_0}$ is given by Equation (21):

$$A_{dil}^{\Omega_0}=A^{\Omega_0}:(A^{\Omega_0}-S^{\Omega_0})^{-1}=[I-S^{\Omega_0}:C_M^{-1}:(C_M-C^{\Omega_0})]^{-1} \quad (21)$$

Expressing the composite average strain as function of the average strain, results:

$$<\varepsilon>=f_I A_{dil}^{\Omega_0}:<\varepsilon>_M+(1-f_I)<\varepsilon>_M \quad (22)$$

Tensor $<\varepsilon>_M$ may be written as function of $<\varepsilon>$, thus:

$$<\varepsilon>_M=\tilde{A}^M:<\varepsilon> \quad (23)$$

where $\tilde{A}^M$ is given by Equation (24):

$$\tilde{A}^M=\left[f_I A_{dil}^{\Omega_0}+(1-f_I)I\right]^{-1} \quad (24)$$

Determining the average strain in the inclusion as function of the average strain in the composite as:

$$<\varepsilon>^{\Omega_0}=A_{dil}^{\Omega_0}:\tilde{A}^M:<\varepsilon> \quad (25)$$

then, using Equations (23) and rearranging terms, results:

$$<\sigma>=f_I C^{\Omega_0}:A_{dil}^{\Omega_0}:\tilde{A}^M+(1-f_I)C_M:\tilde{A}^M \quad (26)$$

where $<\sigma>$ represents the average stress in the composite. Simplifying (26) for $<\sigma>=\overline{C}^{MT}:<\varepsilon>$, where $\overline{C}^{MT}$ is the effective elastic tensor of Mori-Tanaka, given in Equation (27), which leads the determination of the effective properties of composite material.

$$\overline{C}^{MT}=\left[f_I C^{\Omega_0}:A_{dil}^{\Omega_0}+(1-f_I)C_M\right]:\tilde{A}^M \quad (27)$$

4.2 Three-Phase Sphere Model

Based on elasticity theory and the Eshelby equivalent medium theory, [27] developed a three phase sphere model to estimate the effective shear modulus of two phase particulates composite. The three phase sphere model has as hypothesis a sphere of composite material embedded in the infinite medium of unknown effective properties. To determine the elastic properties for this model, the sphere is composed initially by two phases, the matrix (domain Ω) and the inclusion (domain Ω_0), the radii of these two spheres are b and a respectively (Figure 1). The volume fraction of the inclusion is taken as a relation between the radii and is given by

$$f_I=\left(\frac{a}{b}\right)^3$$

On symmetric spherical loading condition, the strain in representative element volume is spherically symmetrical [28], then the bulk modulus is expressed by Equation (28):

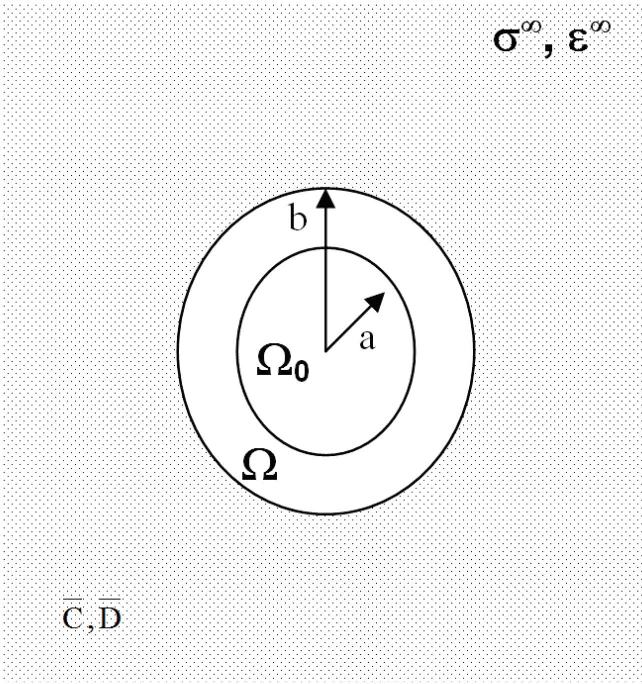
$$\frac{K}{K_M}=1+f_I\frac{(K_I-K_M)(3K_M+4G_M)}{K_M\left(3K_M+4G_M+3(1-f_I)(K_I-K_M)\right)} \quad (28)$$

where K_M and G_M are the bulk modulus and shear modulus for the matrix, while K_I and G_I are the respective modulus for the inclusion and K is the bulk modulus of the composite.
The three-phase model considers the sphere embedded in an infinite and homogeneous medium (Figure 1), submitted to uniform stress and strain applied very distant from the inclusion. These stress and strain are named as σ^∞ and ε^∞, respectively.
The bulk modulus K for this model is the same obtained for three-phase model (Equation 28), for two phases. Hypothetically, it is assumed that the displacement and traction acting on the contour of the body are uniform and linear.
In the case of forced displacement, on the heterogeneous medium boundary, ESHELBY (1956) showed that the strain energy U, under applied displacement conditions, can be determined by Equation (29) [12]:

$$U=U_0-\frac{1}{2}\int_{S_i}\left(\sigma_i u_i^0-\sigma_i^0 u_i\right)dS \quad (29)$$

where S_i is the surface of the inclusion, U_0 is the strain energy in the same medium when it contains no inclusion, σ_i^0 and u_i^0

Figure 1 – Three phase sphere model



are the tractions and displacements in the same medium when it contains no inclusion and σ_i and u_i are the corresponding quantities at the same point in the medium when it does contain the inclusion.
Regarding the composite homogenization process, there is a relationship between the energies of the heterogeneous and homogeneous medium, as presented in Equation (30) and observed in Figure 2:

$$U=U_{eq}\therefore U_{eq}=U_0\therefore U=U_0 \quad (30)$$

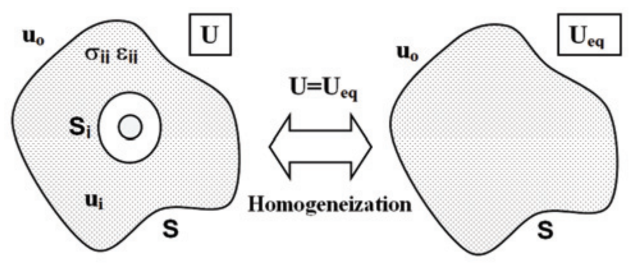
Rewriting the Equation (29):

$$\frac{1}{2}\int_{S_i}\left(\sigma_i u_i^0-\sigma_i^0 u_i\right)dS=0 \quad (31)$$

Applying the stress and strain in the radial and angular directions in Equation (31), results:

$$\frac{1}{2}\int_{S_i}\left(\sigma_r^0 u_{re}+\tau_{r\theta}^0 u_{\theta e}+\tau_{r\phi}^0 u_{\phi e}-\sigma_{re}u_r^0-\tau_{r\theta e}u_\theta^0-\tau_{r\phi e}u_\phi^0\right)dS=0 \quad (32)$$

Figure 2 – Condition of energy balance for the real and homogenized medium



The conditions of shear at the infinite for this problem are given by Equations (33) and (34):

$$\left.\begin{aligned} \sigma_r^0 &= 2GD_1 sen^2\theta \cos 2\phi \\ \tau_{r\theta}^0 &= 2GD_1 sen\theta \cos\theta \cos 2\phi \\ \tau_{r\phi}^0 &= -2GD_1 sen\theta sen 2\phi \\ u_r^0 &= D_1 r sen^2\theta \cos 2\phi \\ u_\theta^0 &= D_1 r sen\theta \cos\theta \cos 2\phi \\ u_\phi^0 &= -D_1 r sen\theta \sin 2\phi \end{aligned}\right\} \quad (33)$$

$$\left.\begin{aligned} \sigma_{re} &= 2sen^2\theta\cos 2\phi\left\{\frac{-3D_1\lambda}{r^3} + G\left[D_1 - \frac{12D_3}{r^5} - \frac{2\left(5-4\bar{\nu}\right)D_4}{\left(1-2\bar{\nu}\right)r^3}\right]\right\} \\ \tau_{r\theta e} &= Gsen2\theta\cos 2\phi\left[D_1 + \frac{8D_3}{r^5} + \frac{4\left(1+\bar{\nu}\right)D_4}{\left(1-2\bar{\nu}\right)r^3}\right] \\ \tau_{r\phi e} &= Gsen\theta sen2\phi\left[-2D_1 - \frac{16D_3}{r^5} + \frac{4\left(1+\bar{\nu}\right)D_4}{\left(1-2\bar{\nu}\right)r^3}\right] \end{aligned}\right\} \quad (34)$$

Evaluating Equation (32) for the effective shear modulus $\overline{G}$, results:

$$A\left(\frac{G}{G_M}\right)^2 + B\left(\frac{G}{G_M}\right) + C = 0 \quad (35)$$

where A, B e C are listed in [11].

4.3 Hashin-Shtrikman Bounds

According to [29], Hashin-Shtrikman developed formulas for the upper and the lower bounds more accurate for the elastic modulus of homogeneous isotropic materials with arbitrary phase geometry by defining a formulation based on the principles of linear elasticity theory.

Conclusions about the Hashin-Shtrikman bounds are: [14]

- If the composite material behave as a continuous two-phase composite, it satisfies the boundaries;
- A truly two phase material must satisfy the bounds. If it does not, it can't be considered as a two phase material;
- The proposition that, if the points are outside of the bounds, the material would not be two phase, implies that it could be assumed as having three or more phases in their constitution.

The lower and upper bounds of Hashin-Shtrikman can be calculated according to Equations (36) and (37), respectively [14, 30]:

$$\begin{cases} K_{\inf} = K_M + \dfrac{f_i}{\dfrac{1}{K_i - K_M} + \dfrac{3f_M}{3K_M + 4G_M}} \\ G_{\inf} = G_M + \dfrac{f_i}{\dfrac{1}{G_i - G_M} + \dfrac{6f_M\left(K_M + 2G_M\right)}{5G_M\left(3K_M + 4G_M\right)}} \end{cases} \quad (36)$$

$$\begin{cases} K_{\sup} = K_i + \dfrac{f_M}{\dfrac{1}{K_M - K_i} + \dfrac{3f_i}{3K_i + 4G_i}} \\ G_{\sup} = G_i + \dfrac{f_M}{\dfrac{1}{G_M - G_i} + \dfrac{6f_i\left(K_i + 2G_i\right)}{5G_i\left(3K_i + 4G_i\right)}} \end{cases} \quad (37)$$

where K_i and G_i are the bulk and shear modulus of the inclusion, K_M and G_M are the bulk and shear modulus of the matrix and f_i and f_M are the respective volume fractions of these phases.

5. Methods

5.1 Experimental Evaluation

According to NBR 8522:2008 [4], the determining process of the concrete tangent elastic modulus must follow a script for the application of loading and also respect the characteristics of measuring devices and measurement bases. The loading process according to this standard, shown in Figure 3, is performed by cycles of loading and unloading searching compatibility of deformations.

The methodology of the test according to NBR 8522:2008 was fixing stress, which evaluates the elastic modulus for a stress about 30% of the specimen rupture stress, previously tested. It is allowed that the upper limit of stress is changed based on any specification. The dispositive used for measuring deformations in the specimens was electric gauges, with a load machine with capacity of 300 tons. The measuring the applied force was carried out by a loading cell with 30 tons capacity. Details of the execution of this test, with the instrumented specimen can be seen in Figure 4. The tests were performed in compression rigid plates (the upper one was articulated) with diameter 150 mm.

Figure 3 – Loading cycles for determining the concrete elastic modulus (NBR 8522:2008)

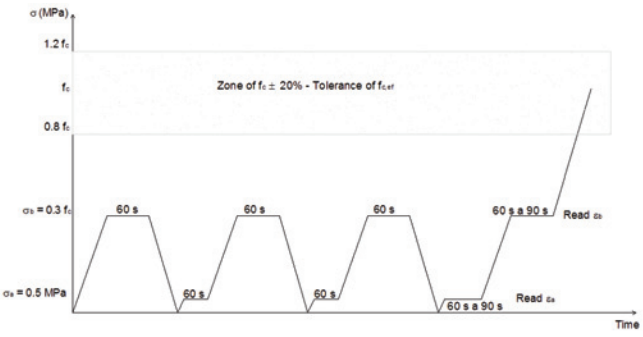


Figure 4 – Instrumented specimen with electrical gauges

For the testing of the elastic modulus a number of three cylinder specimens of 10 cm of diameter and 20 cm of height, for testing at ages 3 to 28 days, were molded. The proportions and other information about mixture are presented in Tables 4 and 5.

Mortar was extracted from each concrete type, coarse aggregate was eliminated by a screening process, and a number of two cylindrical specimens of 5 cm of diameter and 10 cm of height were molded. Both concrete and mortar specimens were capped with cement and sulfur in a ratio 1:3, respectively.

The measurement of the strains was carried out by electrical gag-

Table 4 – Characteristics of aggregates

Properties	Density (kg/m³)	Maximum diameter (mm)	Fine modulus
Fine Aggregate	2560	2.40	2.24
Coarse Aggregate B0	2500	12.50	6.71
Coarse Aggregate B1	2500	19.00	7.96

Table 5 – Mixture proportions of the concrete

Mixture	Proportions in mass (Cement:Sand:B0:B1:Water)	Volume fraction of Cement Paste (f_{pc})
T1	1 : 1.59 : 2.66 : 0 : 0.52	0.326
T2	1 : 1.59 : 0.8 : 1.86 : 0.52	0.326
T3	1 : 1.59 : 2.66 : 0 : 0.52	0.326
T4	1 : 0.99 : 0.6 : 1.4 : 0.39	0.368
T5	1 : 0.99 : 2 : 0 : 0.39	0.368
T6	1 : 0.99 : 2 : 0 : 0.39	0.368
T7	1 : 0.28 : 1.29 : 0 : 0.28	0.487
T8	1 : 0.28 : 1.29 : 0 : 0.28	0.487

Table 6 – Elastic modulus and Poisson's ratio of coarse aggregate

Elastic Modulus	57 GPa
Poisson´s ratio	0.28

es using measurement basis of 100 mm and 50 mm, for concrete and mortar, respectively, as shown in Figure 4.
The results of the determinations of the elastic modulus of the concrete for average values are presented in Table 6.

5.2 Numerical simplifications.

The following simplifications were adopted in the modeling:

- The constituent phases and the effective composite material are assumed to be isotropic, within the linear elastic region.
- The aggregate is considered inert and maintains its elastic properties over time.
- The aggregate is assumed with spherical shape.
- It is assumed that the transition zone between the coarse aggregate and the mortar has constant volume fraction.
- The elastic modulus of the ITZ is assumed to be constant throughout its thickness

The determination of ITZ elastic modulus employed a numerical formulation based on the inversion of the Equation (28) and Equation (35) and the use of experimental results.
The analysis of concrete elastic modulus considering the interfacial transition zone (ITZ) is performed as follows:

- Effective Matrix (Mortar + ITZ) + Aggregate = Concrete. The elastic modulus of mortar + ITZ is obtained by a series model. The series model is described in [3] as: (Equation 38)

$$\frac{1}{E} = \frac{f_1}{E_1} + \frac{f_2}{E_2} \quad (38)$$

6. Concrete Microscopy

For the concrete used in this research a SEM-analysis was accomplished in order to assess the thickness of the ITZ as well as its evolution over time, these results were compared to the information found in the literature.
The tests were performed at the age of 28 days and less in order to verify, through identification of ITZ, if there was variation over time of its thickness. In [31] was observed that the higher porosity that characterizes the transition zone occurs within a minimum of 30 μm. In Figure 5 are presented microphotographs of samples of concrete studied at the age of 28 days. The variable f_{pc} represents the volume fraction of cement paste of the concrete produced.
As a result of the evaluation of these images, it was observed that the thickness of ITZ is not much different between the two types of concrete, ranging from 30 μm to 100 μm, confirming the information presented by the technical literature.

7. Results and Discussion

Elastic properties of the coarse aggregate are given in Table 6. In Tables 7 and 8 are shown the average values of elastic modulus and compressive strength obtained from the lab test described in the NBR 8522:2008 for concrete and mortar, respectively.
Poisson's ratio for mortar was assumed as 0.17 and the elastic modulus of mortar and concrete are shown in Figure 6.
Experimental data were obtained using NBR 8522:2008 and the curves were fitted using polynomial models based on the average values (3 samples each one) for the selected ages. Confidence intervals for small samples were verified by the method of Student t with 95% of significance.
The change on time of concrete elastic modulus according to national codes (listed in section 3), for three selected volume fractions of cement paste, are shown in Figure 7.
From the analysis of code expressions and experimental data it may be noted that:

- The mathematical expression of NBR 6118:2003, for ages of 15 days or more, results in errors up to 8% compared to the experimental data;
- Using the CEB/90 expression it was found better fit, with errors lower than those calculated by NBR 6118:2003 expression;
- The results calculated by ACI/95 and NBR 6118:2003 expressions are similar.
- High errors were obtained by using EUROCODE2/92 expression.

Figure 8 presents the comparative analysis of experimental data by curves generated for the Hashin-Strikman bounds for concrete regarding cement paste volume fractions of 0.326, 0.368 and 0.487.
The Hashin-Shtrikman bounds are used to characterize two-phase composite and, if these limits are exceeded, denotes the presence of one or more extra phases.
It is observed from Figure 8 that almost all experimental points lie outside the bounds proposed by Hashin-Strikman, which implies that the concrete material must be evaluated considering the ITZ as a phase.
The variation over time of the elastic modulus of the Mori-Tanaka and three-phase sphere models in the range of 3 days to 28 days are shown in Figure 9.
The input data to analyze the application of three-phase sphere model are the elastic modulus, Poisson's ratio and volume fraction of phases. The mortar elastic modulus was considered varying over time (Figure 6) and Poisson's ratio of mortar was considered constant.
It is observed that the application of micromechanical models without the transition zone is not satisfactory, showing high errors compared to the experimental fitted curves.
By applying the three phase sphere model, with the inversion of the equations as outlined in the methodology, is determined the ITZ elastic modulus (Figure 9). These curves were obtained by inversion of equations of three phase sphere model, considering the variation of volume fraction of the transition zone in relation to the total mixture.
The curves presented in Figure 10 shows the direct influence of the volume fraction of ITZ and, consequently, its thickness, on the concrete elastic modulus. These values were obtained from the experimental data and show an evolution over time similar to the concrete and mortar.

Making use of the results shown in Figure 10, the concrete elastic modulus has been calculated by direct application of the three phase sphere model according to Equation (38) and it is presented in Figures 11 and 12.

The relationships proposed in the literature refer to the transition zone elastic modulus ranging around 30% to 50% of the matrix elastic modulus, in general, compared to the cement paste matrix. In the analysis considering the matrix made of mortar, which in-

Figure 5 – Microphotographs of the samples of the concrete: (a) f_{pc} = 0.326 – 7 days; (b) f_{pc} = 0.326 – 17 days; (c) f_{pc} = 0.368 – 7 days; (d) f_{pc} = 0.368 – 28 days; (e) f_{pc} = 0.487 – 17 days; (f) f_{pc} = 0.487 – 28 days

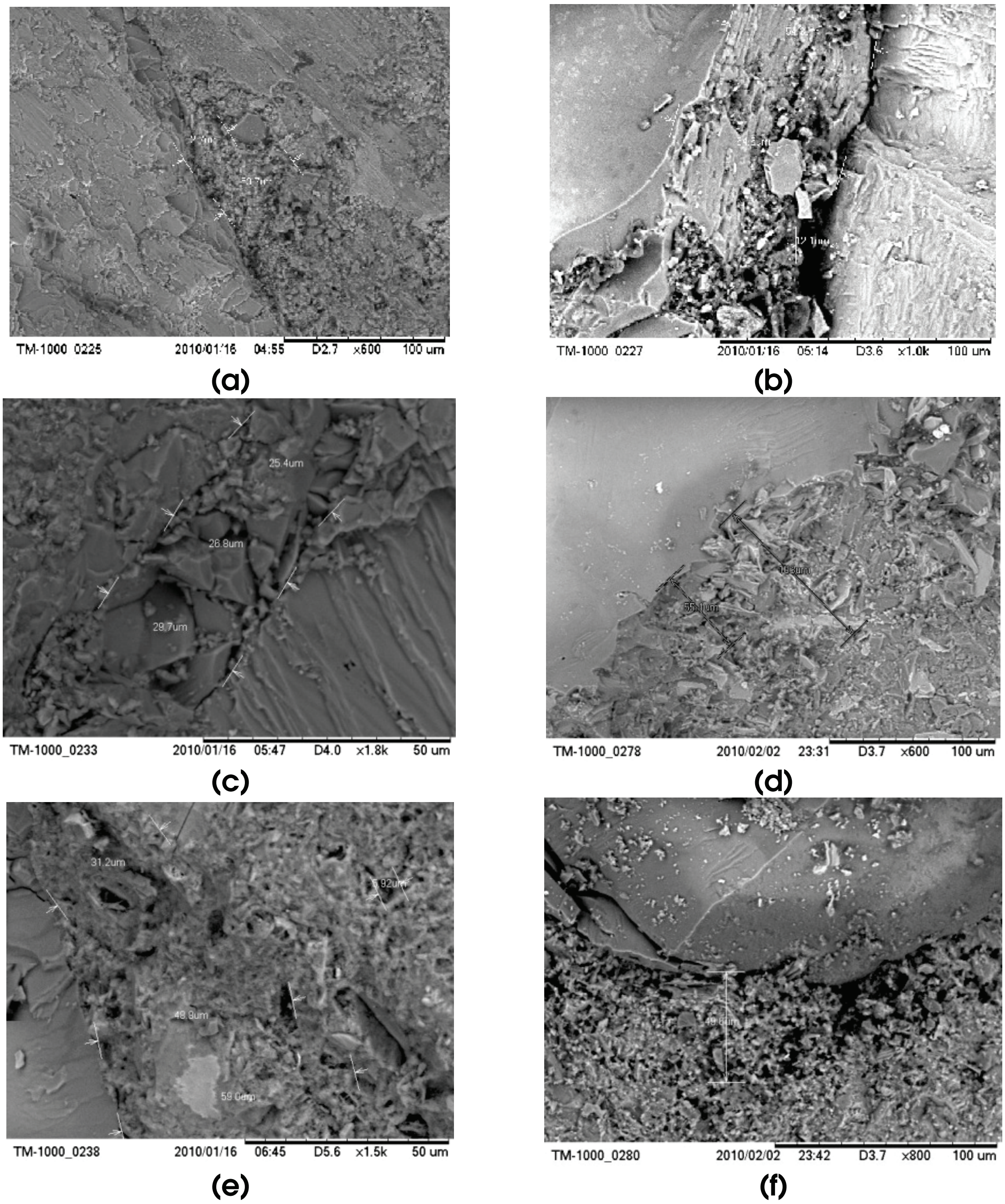


Table 7 – Average values of elastic modulus and compressive strength of the concrete

Results	Elastic Modulus (GPa)				Compressive strength (MPa)			
T1	23.84 (3 days)	27.25 (7 days)	31.87 (15 days)	31.95 (28 days)	13.37 (3 days)	20.40 (7 days)	25.33 (15 days)	27.56 (28 days)
T2	26.31 (3 days)	31.35 (8 days)	30.58 (21 days)	31.91 (35 days)	13.79 (3 days)	19.25 (8 days)	23.89 (21 days)	25.89 (35 days)
T3	27.82 (6 days)	29.63 (20 days)	31.48 (28 days)	- -	22.90 (6 days)	26.14 (20 days)	30.11 (28 days)	- -
T4	33.09 (3 days)	35.83 (7 days)	37.44 (17 days)	34.40 (28 days)	22.26 (3 days)	32.04 (7 days)	33.38 (17 days)	38.12 (28 days)
T5	33.39 (3 days)	36.04 (8 days)	35.70 (21 days)	35.22 (35 days)	24.65 (3 days)	33.46 (8 days)	37.46 (21 days)	41.16 (35 days)
T6	31.33 (6 days)	32.21 (20 days)	32.70 (28 days)	- -	30.02 (6 days)	32.21 (20 days)	42.40 (28 days)	- -
T7	32.56 (3 days)	38.93 (8 days)	35.41 (20 days)	37.13 (34 days)	34.89 (3 days)	40.98 (8 days)	46.07 (20 days)	47.30 (34 days)
T8	31.26 (5 days)	34.99 (8 days)	32.25 (18 days)	36.43 (26 days)	39.07 (5 days)	45.45 (8 days)	47.62 (18 days)	45.96 (26 days)

Table 8 – Average values of elastic modulus and compressive strength of the mortar

Results	Elastic Modulus (GPa)				Compressive strength (MPa)			
T1	16.93 (3 days)	26.00 (7 days)	23.20 (15 days)	27.35 (28 days)	16.68 (3 days)	22.51 (7 days)	30.89 (15 days)	30.37 (28 days)
T2	14.61 (3 days)	23.30 (8 days)	24.29 (21 days)	26.59 (35 days)	17.13 (3 days)	23.42 (8 days)	29.07 (21 days)	27.15 (35 days)
T3	23.63 (6 days)	26.49 (20 days)	27.20 (28 days)	- -	27.22 (6 days)	34.85 (20 days)	31.48 (28 days)	- -
T4	20.29 (3 days)	25.73 (7 days)	29.86 (17 days)	29.51 (28 days)	25.50 (3 days)	34.41 (7 days)	39.05 (17 days)	52.88 (28 days)
T5	27.23 (3 days)	31.09 (8 days)	32.77 (21 days)	31.85 (35 days)	28.50 (3 days)	37.80 (8 days)	42.29 (21 days)	49.21 (35 days)
T6	27.13 (6 days)	29.93 (20 days)	32.12 (28 days)	- -	36.20 (6 days)	46.30 (20 days)	47.52 (28 days)	- -
T7	25.04 (3 days)	31.60 (8 days)	29.92 (20 days)	28.62 (34 days)	33.66 (3 days)	54.08 (8 days)	50.24 (20 days)	55.20 (34 days)
T8	29.86 (5 days)	27.71 (8 days)	28.81 (18 days)	33.12 (26 days)	45.40 (5 days)	52.82 (8 days)	55.33 (18 days)	41.51 (26 days)

Figure 6 – Variation of elastic modulus over time: (a) concrete and (b) mortar

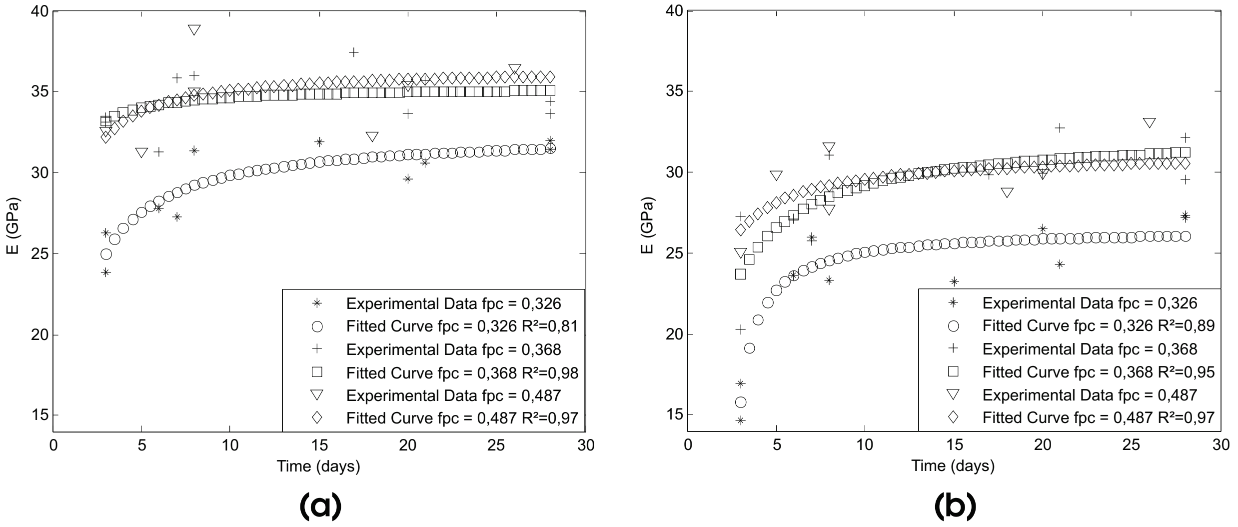
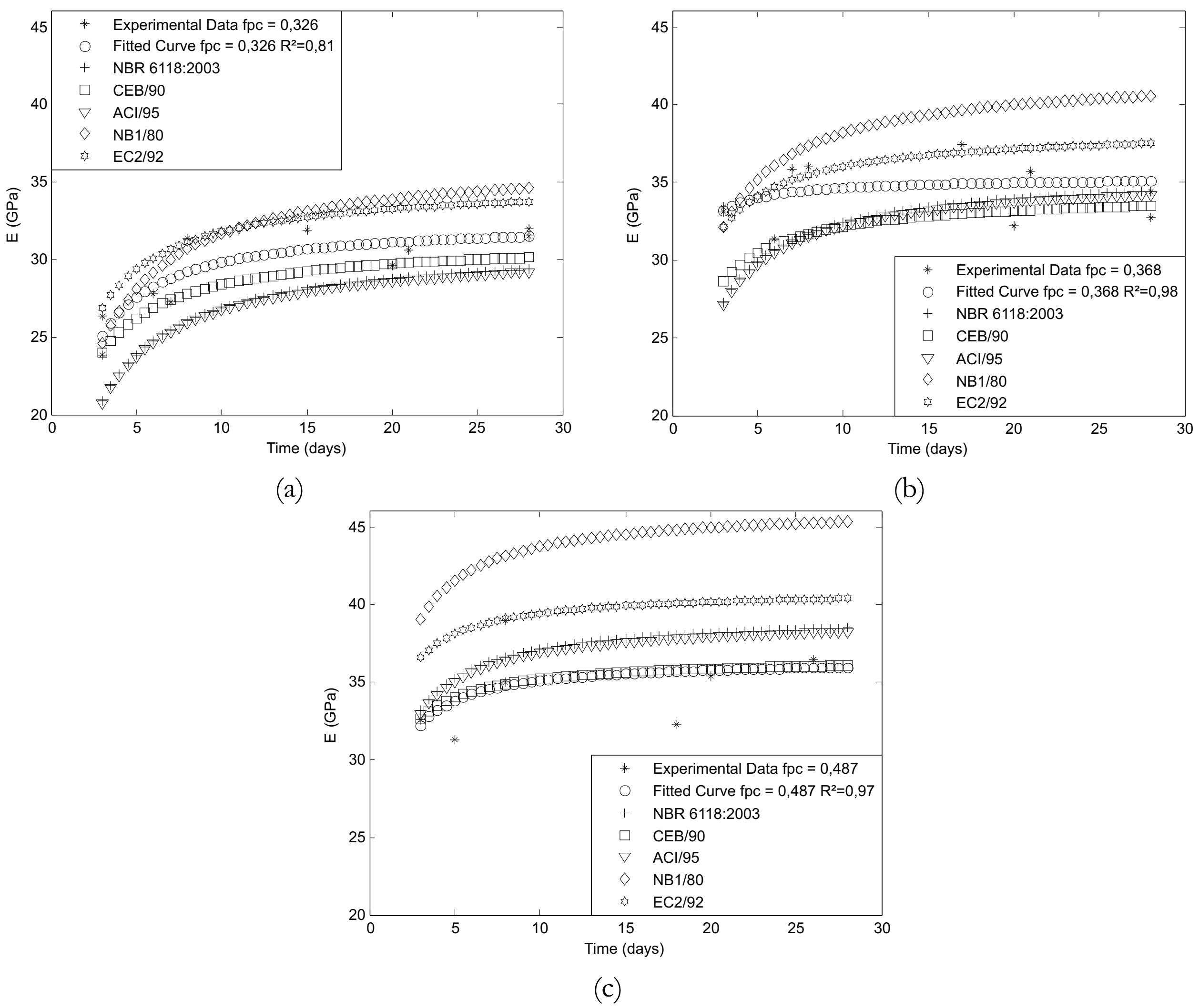


Figure 7 – Comparative study of the evolution over time of the concrete elastic modulus with code expressions: (a) f_{pc} = 0.326, (b) f_{pc} = 0.368 and (c) f_{pc} = 0.487



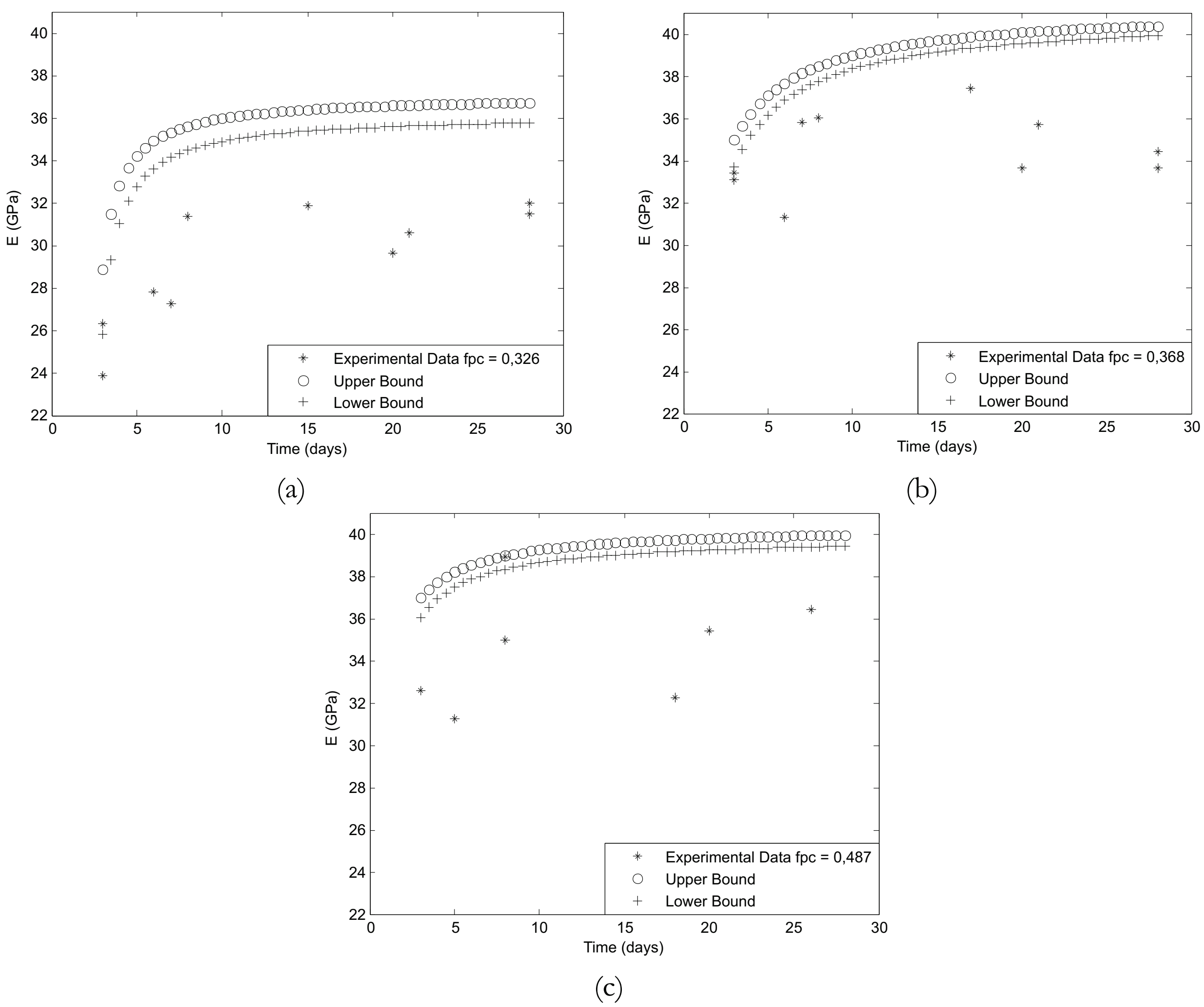
cludes the cement paste and fine aggregate particles, this relationship is in a larger interval than that presented in the references. Additionally, the relationship varies over time because, as the mortar, the transition zone has its strength and rigidity increased over time.

8. Conclusions

The following conclusions may be written:

- The expression proposed by NBR 6118:2003 presents acceptable dispersions in relation to the experimental data obtained, but knowing the relationship with the phase's properties, it could be predicted an adjusting coefficients to better characterize the local concrete in Brazil.
- The CEB/90 expression was the best fitting to the experimental curves, and there were observed similar results for the application in [17] and [22].
- The behavior of the elastic modulus shows a strong dependence on the cement paste volume fraction, which was expected because cement paste has a great influence on the concrete quality. Time in another important parameter because both compressive strength and elastic modulus depends on it.
- The proposition of the elastic modulus as a function of some dosage parameters is important to evaluate the influences of the phases in determining the concrete elastic modulus.
- The relationships obtained for the variation of the ITZ elastic modulus, compared to its matrix of mortar, as well as that obtained for the composite, were within the ranges reported in the literature, but the maintenance of these intervals is conditioned to the volume fraction considered, which provides larger variations. According to the observations made from the numerical

Figure 8 – Hashin-Shtrikman bounds for concrete: (a) f_{pc} = 0.326, (b) f_{pc} = 0.368 and (c) f_{pc} = 0.487



results the transition zone volume fraction is between 10% and 12% and has variable thickness.

- Data reported in the literature show that the ratio of elastic modulus of the transition zone and cement paste varies between 0.3 and 0.5 and, according to the results, this ratio is the same for the mortar and its transition zone.
- The use of transition zone with the matrix of mortar through a series model, when associated to the micromechanical model, showed good results.
- It is possible to conclude on the necessity of consideration of the transition zone for determining the elastic properties of the concrete.

9. Acknowledgments

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Figure 9 – Comparative study of the application of micromechanical models without transition zone: (a) f_{pc} = 0.326, (b) f_{pc} = 0.368 and (c) f_{pc} = 0.487

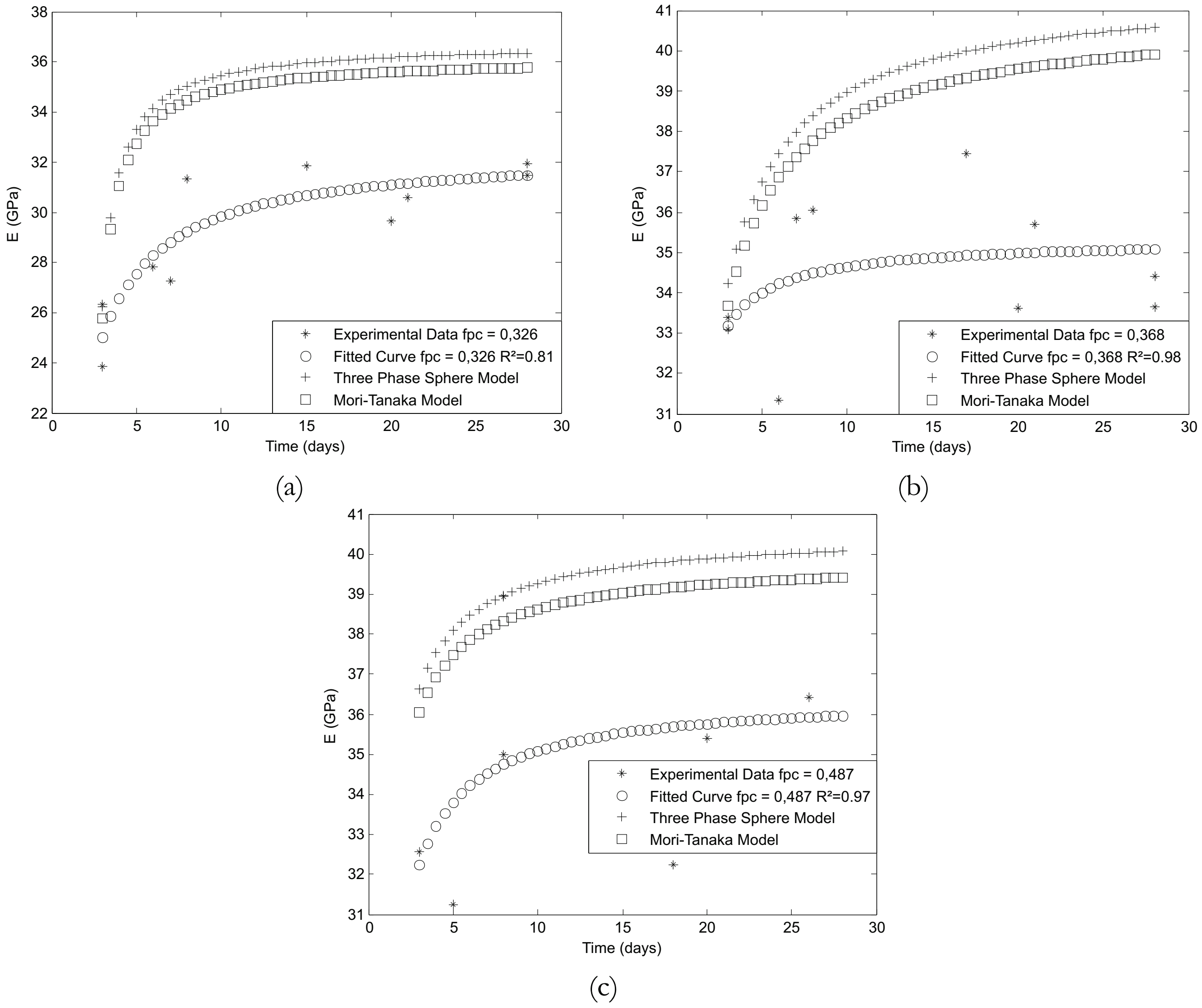

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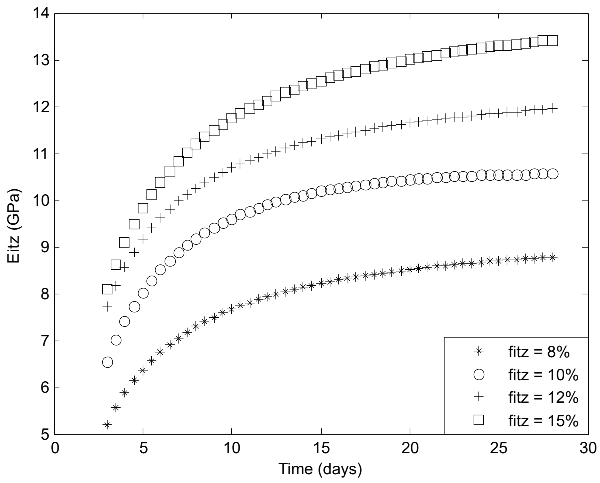
Figure 10 – Comparative study of the transition zone elastic modulus to the concrete: (a) f_{pc} = 0.326, (b) f_{pc} = 0.368 and (c) f_{pc} = 0.487



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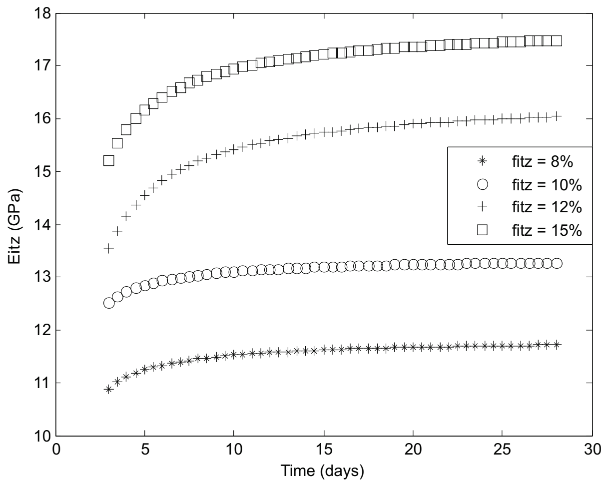

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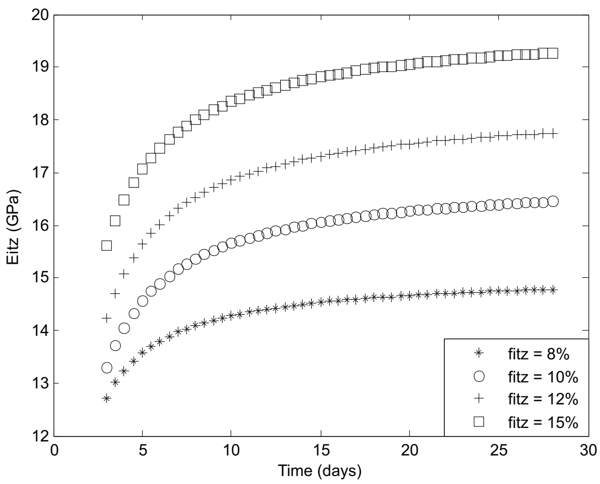

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Figure 11 – Comparative study of the concrete elastic modulus obtained with three phase sphere model: (a) f_{pc} = 0.326, (b) f_{pc} = 0.368 and (c) f_{pc} = 0.487

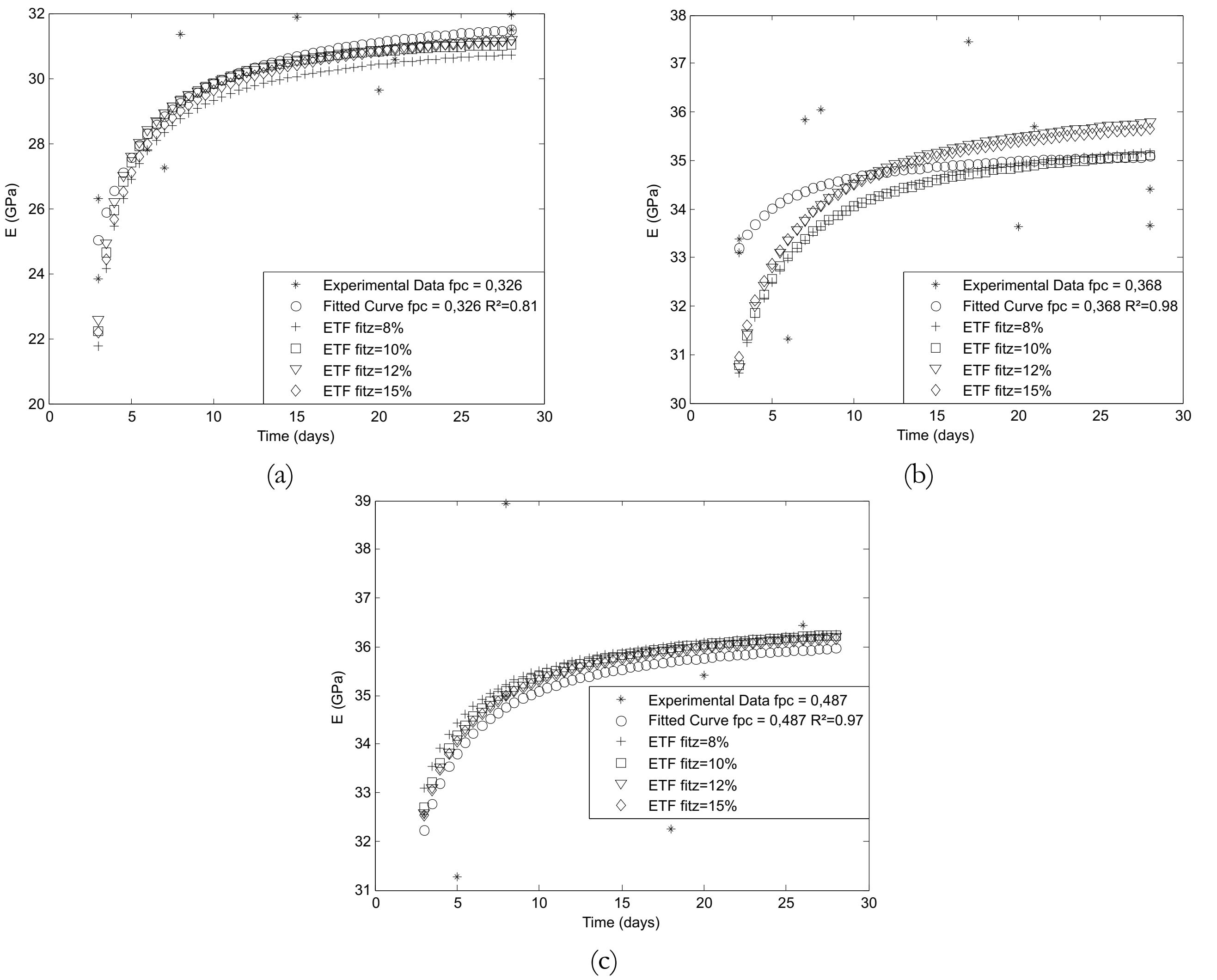

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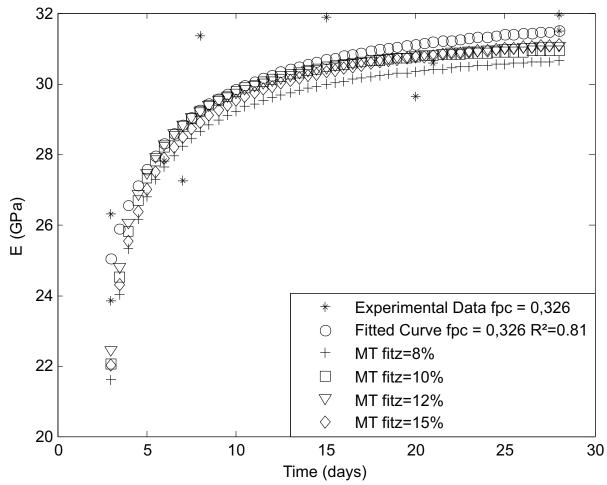
Figure 12 – Comparative study of the concrete elastic modulus obtained with Mori-Tanaka model: (a) f_{pc} = 0.326, (b) f_{pc} = 0.368 and (c) f_{pc} = 0.487



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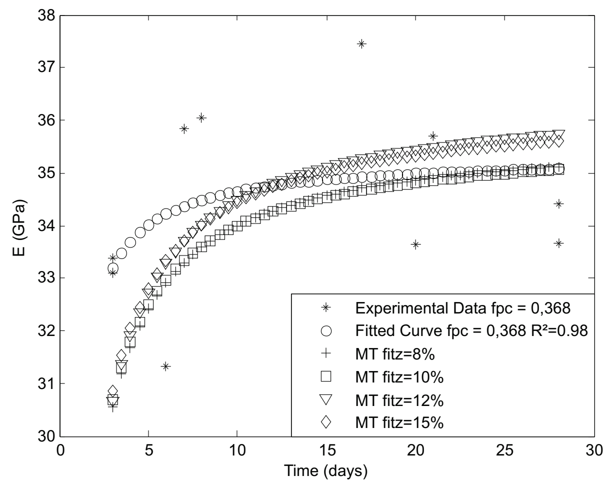

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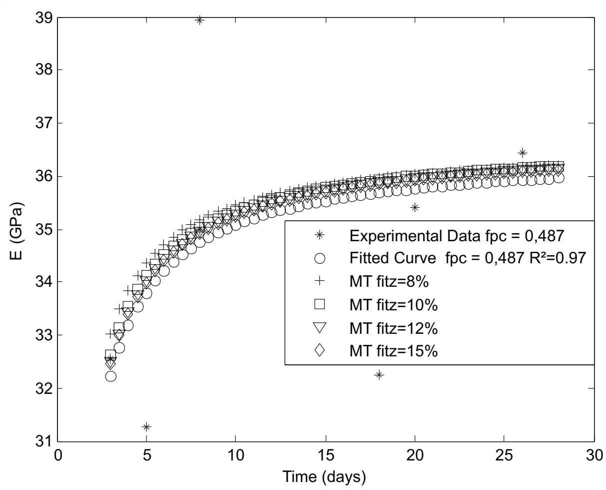

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