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Fully probabilistic design – the way for optimising of concrete structures

Diseño probabilístico completo – como optimalizar las estructuras de hormigón





I. LANÍKOVÁ [a]
lanikova.i@fce.vutbr.cz

P. ŠTĚPÁNEK [b]
stepanek.p@fce.vutbr.cz

P. ŠIMŮNEK [c]
simunek.p@fce.vutbr.cz

D. HORÁK [d]
horak.d@fce.vutbr.cz

M. ZLÁMAL [e]
zlamal.m@fce.vutbr.cz

Abstract

Some standards for the design of concrete structures (e.g. EC2 and the original ČSN 73 1201-86) allow a structure to be designed by several methods. This contribution documents the fact that even if a structure does not comply with the partial reliability factor method, according to EC2, it can satisfy the conditions during the application of the fully probabilistic approach when using the same standard. From an example of the reliability of a prestressed spun concrete pole designed by the partial factor method and fully probabilistic approach according to the Eurocode it is evident that an expert should apply a more precise (though unfortunately more complicated) method in the limiting cases. The Monte Carlo method, modified by the Latin Hypercube Sampling (LHS) method, has been used for the calculation of reliability. Ultimate and serviceability limit states were checked for the partial factor method and fully probabilistic design. As a result of fully probabilistic design it is possible to obtain a more efficient design for a structure.

Keywords: probabilistic design, partial reliability factor, concrete structures design.

Resumo

Algúnos estándares para el diseño de estructuras de hormigón (p. ej. EC2 y la original ČSN 73 1201-86) permiten que las estructuras sean diseñadas utilizando diferentes métodos. Esta contribución trata casos en los cuales una estructura dada que incluso no obedece el método de índice de confiabilidad parcial, de acuerdo a EC2, puede satisfacer las condiciones requeridas durante la aplicación del método probabilístico cuando se usa el mismo estándar. Tomado de una muestra de la confiabilidad de un hormigón pretensado hilado diseñado con la ayuda del método de índice parcial y por el método probabilistico de acuerdo al código Euro (Eurocode) es evidente que un experto debe aplicar un método más preciso (pero desafortunadamente más complicado) en los casos limitantes. El método Monte Carlo modificado por el método (LHS) Latin Hybercube Sampling ha sido utilizado para efectuar el cálculo de confiabilidad. Los límites extremos y de utilidad fueron comprobados para el método de índice parcial y diseño probabilístico completo. Como resultado del método de diseño probabilístico es posible obtener un diseño más eficiente para una estructura.

Palavras-chave: diseño probabilístico, índice de confiabilidad parcial, diseño de estructuras de hormigón.

[a] *Brno University of Technology, Faculty of Civil Engineering, lanikova.i@fce.vutbr.cz, Veveří 95, 602 00 Brno, Czech Republic*
[b] *Brno University of Technology, Faculty of Civil Engineering, stepanek.p@fce.vutbr.cz, Veveří 95, 602 00 Brno, Czech Republic*
[c] *Brno University of Technology, Faculty of Civil Engineering, simunek.p@fce.vutbr.cz, Veveří 95, 602 00 Brno, Czech Republic*
[d] *Brno University of Technology, Faculty of Civil Engineering, horak.d@fce.vutbr.cz, Veveří 95, 602 00 Brno, Czech Republic*
[e] *Brno University of Technology, Faculty of Civil Engineering, zlamal.m@fce.vutbr.cz, Veveří 95, 602 00 Brno, Czech Republic*

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1. Introduction

Most of the standards enable the use of a variety of methods for the design of a particular structure. The obtained outcome of these methods depend on

- the level of simplification of the calculation within a particular procedure,
- the quality of the input data,
- the expertise of the engineer, and the amount of time he has available to spend on the structural design.

The partial reliability factor method is presently mostly used for the design and review of a structure from the standpoint of ultimate states. This method is used especially for its simplicity and the ease of obtaining input data. The input data result from the used materials and loads (relating to the structure's function and its location). Characteristic input data values are given in relevant parts of the standards and drawing documentation. However, standard [1] alternatively also enables the application of probabilistic methods for the design and review of structures. Basic information is mentioned in [1] and [2].

2. Design methods

The design process includes lot of uncertainties, especially

- the randomness of physical quantities used in the design (as a natural characteristic of each quantity),
- statistical uncertainties during the description of a particular quantity caused by a lack of data,
- model uncertainties caused by inaccuracies in the calculation model in comparison with real structural behaviour,
- uncertainties caused by inaccuracies in the limit state definition,
- mistakes and human element deficiencies within the design procedure, execution, maintenance and usage of the structure, along with incomplete knowledge of real material and structural behaviour.

In the classic approach of evaluating the reliability of a structure using the partial reliability factor method, the first three above-mentioned groups of uncertainties are hidden within in the partial reliability factors, which are determined separately both for the effects of load E and structural resistance R. During the application of this approach it is not necessary to know the particular "values" of the uncertainties. The reliability review is simplified within the observance of rules and recommendations but the real probabilistic base of the reliability review stays hidden.

The ultimate limit state (ULS) reliability condition is

$$R_d \geq E_d \tag{1}$$

and for serviceability limit states (SLS)

$$C_d \geq E_d \tag{2}$$

where

R_d is the design value of structural resistance,
C_d is the design value of the relevant criterion of serviceability,
E_d is the design value of the effect of loading.

The values of individual partial reliability factors depend upon the type of assessed limit state and the reliability category. These values are presented in [1] and in the appropriate EC standards and national appendixes.

The characteristic values of the material properties represent either the mean values (e.g. modulus of elasticity) or 5%, 95%, or alternatively a different distribution fractile of the considered random quantity (e.g. strength characteristics, limit strain) with a dependence on the type of limit state and assessed reliability condition.

The probabilistic formulation procedure expressing structural reliability [4] considers the variable quantities entering the calculation to be random quantities whose uncertainties can be described by means of mathematical statistical methods. This approach requires either knowledge of the probability distribution of these quantities or at least knowledge of the distribution of the statistical parameters, and possibly the mutual statistical dependence or independence of individual quantities. The reliability condition is usually expressed with the help of failure function Z

$$Z = g(R, E) = R - E \tag{3}$$

A value of $Z \geq 0$ means a failure-free state (reliability reserve); a value of $Z < 0$ means a structural failure. The quantities E (effect of loading) and R (structural resistance) are random quantity functions that represent in principle the geometrical and material characteristics, load, and possibly the influences of other factors – see Figure (1). The failure probability can be determined (e.g. [5]) in the form.

$$p_f = P(R < E) = P(Z < 0) = \int_{Z<0} f_Z(z)dz \tag{4}$$

Figure 1 – Random variables: structural resistance R, effect of loading E

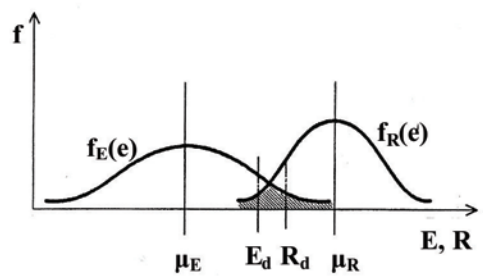
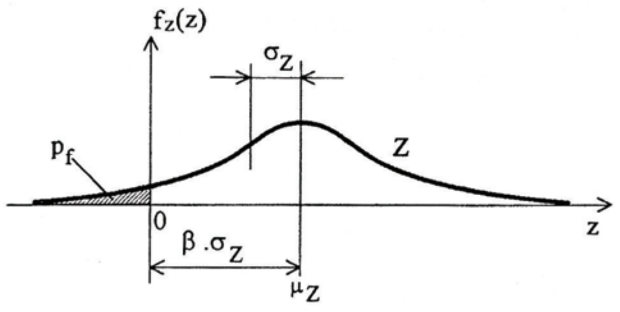


Figure 2 – Failure function Z, failure probability p_f, reliability index β



where $f_Z(z)$ is the density of the failure function distribution probability – see Figure (2).
The reliability condition is expressed as

$$p_f \leq p_0 \tag{5}$$

where p_0 is the target value of the structural failure probability.
If the failure function Z has normal distribution with the parameters μ_Z (mean value) and σ_Z (standard deviation), it is possible alternatively to use reliability index β as a reliability indicator according to the relation

$$\beta = \frac{\mu_Z}{\sigma_Z} \tag{6}$$

where it is valid that

$$p_f = P(Z \leq 0) = P(g \leq \mu_Z - \beta\sigma_Z) \tag{7}$$

The reliability condition is then in the form

$$\beta > \beta_0 \tag{8}$$

where β_0 is the target value of the reliability index related to different design situations and the referential time for load-bearing members with a dependence on the reliability category given in [1] and [2].
However, the failure probability determined in this way only represents a hypothetical failure level and generally does not correspond to the real failure probability. This approach involves about 20% of the total number of failures. Other uncertainties caused by construction, operation and other influences [6] are not involved in the calculation and form the main focus of interest in the field of risk engineering.
Analytical representation of the failure function is only possible in simple cases, and thus its application is limited. The failure function $Z = g(R, E)$ generally depends on the number of random quantities, whose distributions do not always correspond to the normal distribution. The relations for determining the effect of loading E and resistance function R are often complicated and non-linear; therefore, numerical methods (simulation, semi-analytical) are used for failure probability calculation.
The simulation methods are based on a simulation of the particular realizations of a random vector. The Monte Carlo method is closest to engineering requirements. Discrete values of the resistance function R and the effect of loading E, possibly failure function Z, are calculated for N generated realizations of the random selection. The failure probability can be easily determined from the ratio of the number of realizations N_f, when a failure occurs ($Z \leq 0$) to the total number of realizations N. High demands in terms of the amount of simulations at low values of estimated failure probability are a disadvantage of this method.
A modification method, e.g. the Latin Hypercube Sampling method (LHS), is suitable for the solution of extensive tasks when individual calculations are time-consuming. The reliability index is determined from statistical parameters of the failure function, which are determined again from a certain number N of realized random quantities. This method provides very good estimations of the failure function in comparison with the classical Monte Carlo method even at a low number of simulations because it ensures that the whole range of the random quantity is uniformly processed in reference to its distribution function.
Simulation results can be combined with new measurements and improved by using Bayesian techniques, see, e.g. [7]. An overview of numerical techniques applicable to large-scale optimum design problems involving uncertainty can be found in [8] and they are applied to design problems in [9].

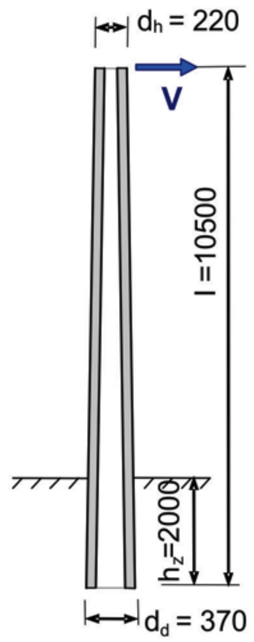
3. Object of studies

Reinforcement for use in a pole made from spun concrete - see Figure (3) - was designed using partial reliability factor method in accordance with the principles of standards [1] and [3] in order to meet the following reliability condition requirements:

- ULS within normal force load (caused by prestressing) and the bending moment (caused by the top horizontal force V) in predefined discrete cross-sections along the pole depth,
- SLS controlling
- deflection of the pole head due to the top horizontal force V,
- crack development at 0.5V load,
- crack widths for 1.0V load.

Design and assessment were carried out with the software based on the algorithms determined and mentioned in [10] and [11] using the partial reliability factor method. 20 pieces of the PN6 (prestressing indented wire 6 mm in diameter) designed for the critical section are distributed in one layer around the pole perimeter and 17 pieces of the R10 (passive reinforcement B500B 10 mm in diameter) are uniformly distributed into the interspaces between prestressing wires. The prestressing wires are situated along the entire pole and are bond-anchored both at the top and bottom of

Figure 3 – Geometry of the pole



the pole. Passive reinforcement is designed in three lengths in order to contribute to the bearing capacity of the pole.

A review of the pole was done in agreement with the USL and SLS with the help of the fully probabilistic method. Alternative designs with a low number of bars and wires were also created. Only some variables were considered within the frame of calculation, namely those with a dominant influence on structural reliability:

- **the material characteristics of**
  - concrete – compression and tensile strength, modulus of elasticity, ultimate strain of compressed concrete,
  - non-prestressing bars – yield strength, ultimate strength, tensile ultimate strain, reinforcement area,
  - prestressing wires – ultimate strength (proof limit 0.1), modulus of elasticity, tensile ultimate strain,
- **geometrical characteristics, such as**
  - foundation depth (fixation),
  - reinforcement cover,
- **horizontal loading by the top force V.**

The other quantities were considered as deterministic. The statistical parameters of most of the chosen random quantities were taken from the recommendation of the Joint Committee on Structural Safety [12] so that the resulting distribution functions of these random quantities correspond as much as possible to the real situation. The stress acting on the pole is given by the top horizontal force. The characteristic value of this force is V = 10 kN. It includes the tensions of cable lines and also climatic effects (such as wind and icing on overhead lines). The distribution type was considered lognormal with mean μ = 5.5 kN and standard deviation σ = 2.75 kN (C.o.V. = 0.5). It is assumed that these statistical parameters are determined for the total service life of the pole.

In accordance with the recommendation of the JCSS [12], the uncertainties of the resistance model and the calculation of the effect of loading E should also be included in the reliability calculation by the means of random variable θ_R and θ_E .

500 numerical simulations were performed using the LHS method. For each calculation a vector of the random quantities was individually generated using FREeT [13] computer software. The virtual numerical simulations provided function value files for the effect of loading E, structural resistance R and failure function Z for reviewing by the ULS. Others values, such as the top deflection, the development of cracks and their propagation along the pole depth were used for reviewing the structure in terms of the SLS. These quantity distribution parameters were estimated with the help of statistical methods and the failure probability was determined together with the reliability index.

Only a critical section at the point of fixation was assessed by the fully probabilistic method, from the point of view of the ULS. The reliability index according to relation (6) for the distribution of normal failure function Z with the parameters N(162.0; 39.43) is β = 4.11, and the failure probability is $p_f = 1.99 \cdot 10^{-5}$. Within the Eurocode [1] the pole can be classed as reliability class RC2, for which a minimal reliability index is recommended for the ULS as well as a referential time of 50 years β = 3.8. Thus, it is evident that its structure can be designed more economically when using the fully probabilistic method – see Fig. 4. From the graph it is evident that for predefined statistical parameters of designed quantities, the assessed section of the pole will satisfy the ULS criteria even with a lower amount of reinforcement. For example, by reduction of the passive reinforcement from 17 to 13 pieces and by keeping the same amount of active reinforcement (20 pieces), the reliability index decreases to a value of 3.85. All of the reinforcement designs whose curves are presented in the graph above the reliability index value β = 3.8 (marked by a dashed line) will satisfy the criteria.

With regard to the specific shape of a section (annulus) and regular reinforcing along the pole circumference, even at a lower value

Figure 4 – Reliability index of resistance

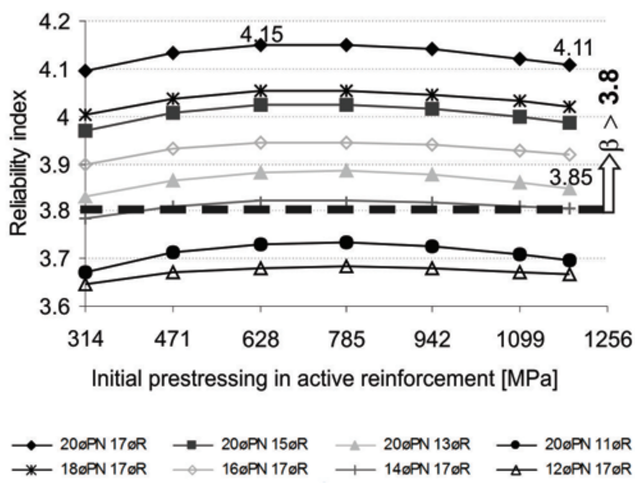
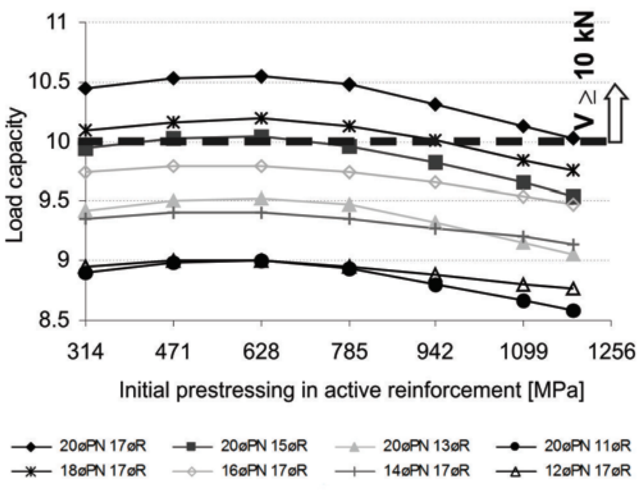


Figure 5 – Load capacity



of initial stress in prestressing reinforcement, the resulting value of the reliability index is higher (for $\sigma_{p,max}$= 628 MPa is β = 4.15) than for the maximal allowable stress (for $\sigma_{p,max}$= 1188 MPa is β = 4.11) to which structures are usually prestressed – see the graph in Figure (4). This means that a reserve of load-bearing capacity at the low value of initial stress is higher then for the maximal allowable stress in the prestressing reinforcement. Figure (4) also displays records of the dependence of the reliability index on the initial prestressing for the different calculations of both the active and passive reinforcement.

Figure (5) shows the load capacity of a pole stressed by vertex tension determined by the partial reliability factor both for the given reinforcement and the magnitude of the initial prestressing in the active reinforcement. The structure was designed for a characteristic vertex tension value of 10 kN. The boundary is marked by a dashed line.

All designed poles which comply with bearing resistance by fully probabilistic method are reviewed with the SLS by the same method.

The deflection limit is given by 4% from the height of the pole above the ground, i. e. 0.340 m. The minimal corresponding reliability index is β = 3.22, which fulfils the reliability condition for a reversible serviceability limit state (reliability class RC2 and referential time 50 years) that is given by the value β_0 = 0 (p_0 = 0.5). According to the fully probabilistic design method, all designed poles comply with this condition.

Crack origin is an irreversible serviceability limit state for which a value of reliability of β_0 = 1.5 is prescribed with a corresponding probability of failure development of $p_0 = 6.68{\cdot}10^{-2}$. Only poles with 18 ø PN6 or 16 ø PN6 do not fulfil this reliability condition.

From the viewpoint of the crack widths it is supposed that it is an irreversible ultimate state of serviceability for which the ultimate value of failure probability $p_0 = 6.68{\cdot}10^{-2}$ is prescribed. All designed poles satisfy this condition (poles with the initial prestressing in the active reinforcement $\sigma_{p,max}$ = 1188 MPa).

These results are comparable with assessment by partial reliability factor method. It is evident that the reliability of the SLS review determined by both methods is approximately the same, and no significant differences are demonstrated.

4. Conclusions

From the presented study it is evident that both of the mentioned methods provide different levels of design reliability. The maximum difference between the results was obtained in terms of the ULS review. Comparable results were gained within the scope of all serviceability limit states. However, fully probabilistic design requires, compared with the partial reliability factor method,

- knowledge of the definition of the distribution of input quantities,
- a design software (algorithm) for re-calculation,
- sufficient professional knowledge.

The application of the fully probabilistic design approach and the verification of reliability enables the introduction of mass production with the option of control to the design procedure, and increases in quality. Individual input quantities which are considered as random quantities with a given probability distribution (or statistical distribution parameters) can be gained by long-term monitoring and evaluation.

Input quantities significantly influencing structural reliability can be determined with the help of parametric testing of the influences of the individual input quantities on the resulting values of failure probability in terms of ULS and SLS. This means there is a possibility of feedback on the designs of structures for production, and that the desired improvements in quality will occur. Efficient design procedures can achieve not only cost savings during construction (materials and energy), erection, servicing, maintenance, disassembly and material recycling but also a more favourable environmental impact.

Determination of the number of simulations in the course of a predefined interval estimation of probability of the occurrence of the monitored feature (e.g. failure probability) is a problem to be solved in the future.

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