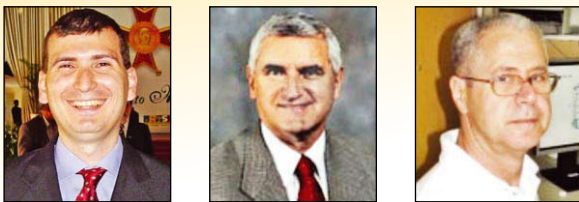


Development of truck weight limits for concrete bridges using reliability theory

Desenvolvimento de equações para a limitação do peso de veículos de carga em pontes de concreto através da teoria de confiabilidade



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Abstract

The increase in gross weight limits allowed by Brazilian legislation and the soaring number of new truck configurations on national highways has called for greater attention regarding the structural safety of bridges when submitted to real traffic. This paper verifies the performance of bridges under Sao Paulo Department of Transportation jurisdiction using the reliability index β and obtains truck weight limits in order to guarantee structural integrity. The superstructures of reinforced and prestressed concrete bridges, classes 36 and 45, are considered. The ultimate limit state is verified in cross sections subjected to positive and negative critical bending moments. In case of prestressed bridges, the cracking limit state in concrete is added. The real traffic is represented by a live load model based on weighting data collected from stations located on highways of the state of Sao Paulo and the statistical resistance parameters are determined using the Monte Carlo technique. The gross weight limits are presented in the form of equations known as bridge formulas which are applicable to any group of two or more consecutive axles. The observed results indicate restrictions to the traffic of some vehicles, especially the 740 kN and 19.80 meters length truck. Considering only the serviceability limit state, class 45 bridges are found to exhibit lower weight limits due to the load factors recommended by the code during design.

Keywords: concrete bridges; reliability theory; live load model; bridge formula; truck weight regulations.

Resumo

O aumento nos limites de peso estabelecidos pela legislação brasileira e a proliferação das Combinações de Veículos de Carga nas rodovias nacionais motivam uma preocupação no que se refere à segurança estrutural das pontes quando submetidas ao tráfego real. Este trabalho verifica o desempenho das obras-de-arte especiais sob jurisdição do DER-SP através do índice de confiabilidade β e obtém equações para a limitação do peso de caminhões de modo a não comprometer sua integridade estrutural. São consideradas as superestruturas das pontes em concreto armado ou protendido, classes 36 e 45. Verifica-se o estado limite último nas seções transversais mais solicitadas por momento fletor positivo e negativo. No caso de pontes em concreto protendido, acrescenta-se a verificação do estado limite de formação de fissuras. O tráfego real é representado por um modelo de carregamento móvel baseado em pesagens de caminhões efetuadas em rodovias do estado de São Paulo e os parâmetros da resistência são determinados através da técnica de Monte Carlo. Apresenta-se os limites de peso em forma de equações, denominadas ECPLs (Equações Comprimento-Peso Limite), aplicáveis a quaisquer grupo de eixos consecutivos dos veículos. Os resultados indicam restrições à circulação de algumas composições, especialmente ao rodotrem de 740 kN e 19,8 metros de comprimento. Considerando-se apenas o estado limite de serviço, as pontes classe 45 apresentam menores limites de peso devido à ponderação de ações durante o projeto.

Palavras-chave: pontes de concreto; teoria da confiabilidade; carga móvel; combinações de veículos de carga.

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1. Introduction

Efficiency in the transportation segment is certainly one of the requirements for the economical development of emerging countries. Particularly concerning to road transportation, it is interesting that a nation has a modern fleet of trucks capable of transporting great amounts of load without pressure on the cost of goods.

In that context, Brazil has recently seen the growing use of Long Combination Vehicles (LCVs) in substitution to old unitary vehicles or vehicles with just a towed unit. There are no doubts on the benefits resulting from such an innovation in course. However, it is necessary that the existing legislation be adapted to suit the new reality and that rules be defined to be effectively accomplished. Besides government agencies, the subject also directly interests private administrators of public highways responsible for the appropriate maintenance of the segments under their responsibility. One of the aspects to be carefully analyzed is the eventual damages that the existing structures may suffer due to the circulation of the new truck configurations. The lack of appropriate maintenance and the consequent state of deterioration of some bridges worsen that scenario. It is therefore necessary to evaluate the consequences of heavy vehicles traffic and to define efficient and reliable criteria that aid in the decision making as to which configurations of LCVs, especially short and heavy, are permitted to use certain segments of the Brazilian road mesh.

The study of the effects of LCVs on the most common structural systems in highways of the state of Sao Paulo began at the Department of Structural Engineering of the Sao Carlos School of Engineering through El Debs et al. [1]. That report, based on the comparison between load effects due to LCVs and the normative load indicated restrictions to the traffic of some vehicles and served as guide in the elaboration of rules for issuance of special permits from Sao Paulo Department of Transportation. Later, two other reports (El Debs et al. [2] and [3]) with similar objectives were published, thus evidencing the need of a procedure that can be applied in a more generic way without the structural analysis of several bridges for each individual truck.

In October of 2005, the National Council of Traffic approved Resolution n° 184, that increased the maximum gross weight of LCVs

with two or more units (included the tractor unit) from 450 kN to 570 kN, avoiding the need of a special permit for those compositions and ratifying decision already considered in Resolution n° 164 of September, 2004. It was also determined that LCVs with a total gross weight higher than 570 kN and lower or equal to 740 kN should have a length equal to or higher than 25 meters, but not longer than 30 meters. As from January 2007, the weight and dimensions limits were defined by Resolution n° 210 (that revoked Resolution n° 12) while the necessary requirements for the circulation of LCVs were determined by Resolution n° 211 (that revoked Resolutions n° 68, 164 and 184).

It should be emphasized here that the consideration of only the gross weight turns out to be unable to supply indications on the safety of a bridge. The relationship between gross weight and applied load length is essential for the correct understanding of the problem.

One of the alternatives to be followed comes from American legislation which, besides imposing maximum limits for isolated axles at 89 kN, axles in tandem at 151 kN and the total weight of vehicles at 356 kN, states that the weight of a group of two or more consecutive axles is restricted according to the so-called bridge formula or truck weight formula given by:

$$W = 2.224 \cdot \left[\frac{B \cdot N}{0.3048 \cdot (N - 1)} + 12 \cdot N + 36 \right] \quad (1)$$

where

W: overall gross weight in kN on any group of two or more consecutive axles;

B: length in meters of the axle group;

N: number of axles in the axle group.

Although considered excessively restrictive, equation [1], besides presenting a very convenient approach to determining the maximum weight as a function of the distance between axles groups, it is also of easy application. Using that original idea and introducing probabilistic processes, the above equation can be improved thus indicating a realistic maximum load to which existing bridges can be safely subject (Ghosn [4]).

Table 1 – Selected cases for analysis

Structural system	Simply supported				Continuous
	10	20	30	40	
Slab	RC	RC / PC	-	-	RC
2 girders	RC	RC	-	-	RC
5 girders	RC	RC / PC	PC	PC	RC
Box section	-	RC / PC	PC	PC	RC

PC: Prestressed concrete | RC: Reinforced concrete

Table 2 – Load factors for ultimate limit state

Period	Until 1977	1978 to 1983	1984 to 1986	1987 to 2002	Starting from 2003
Class	36	36	45	45	45
γ_g	1,65	1,40	1,40	1,35	1,30
γ_q	2,00	1,68	1,40	1,50	1,50

γ_g : Load factor for dead load | γ_q : Load factor for live load

Using the reliability theory, the objective of the present paper is to quantify the safety of cross sections subjected to a real traffic representation and to propose truck weight limits in reinforced and prestressed concrete bridges of classes 36 and 45. The presented procedure is based on Ferreira [5], where more details can be found.

2. Resistance of cross sections

The determination of the statistical parameters of cross section strength involves the selection of typical bridges, their modeling and analysis of internal effects, their design to bending moment, the calculation of the nominal bending moment resistance and the implementation of Monte Carlo's technique. The procedure includes the analysis of the compressive strength of concretes with several design specifications.

2.1 Typical bridges

The bridges considered in this study were chosen based on the available material supplied by Sao Paulo Department of Transportation, already used in El Debs et al. [1], in a total of 60 bridges. According to the structural system, they are:

- slab: 8 bridges;
- 2 girders: 23 bridges;
- 5 girders with crossbeam: 11 bridges;
- 5 girders without crossbeam: 8 bridges;
- box section (1, 2, 3 ou 4 cells): 10 bridges;

The structural analysis was carried out using the software STRAP (Structural Analysis Programs) - version 9.0 (SAE Informatica[6]).

2.2 Design

Once the bending moment is obtained, the determination of the areas of the longitudinal steel is carried out for each cross

section used in the safety analysis. For the total number of structures analyzed, some are considered only in reinforced concrete, some only in prestressed concrete while some apply to both cases. In the case of simply supported bridges, except 2 girder bridges, for 10 m spans reinforced concrete was adopted, for 20 m spans, reinforced and prestressed concrete, and for spans longer than 30 m only prestressed concrete. In the case of continuous structures and for 2 girder bridges, only reinforced concrete was considered. A summary of the selected cases is given in Table 1.

Due to changes in Brazilian codes along the years, several periods are considered both for reinforced and prestressed concrete. Each period corresponds to a specific safety coefficient and calculation assumptions.

2.2.1 Reinforced concrete

Considering the evolution of design methodology, five periods were determined, two being for class 36 and three for class 45. The load factors associated to each period may be seen in Table 2.

The values adopted for the characteristic strength of concrete are given in Table 3. Although that strength is a variable parameter according to each design, representative values were adopted for each period. It should be pointed out that the same characteristic strength is adopted for all bridge types.

2.2.2 Prestressed concrete

According to information obtained from experienced professionals, limited prestressing is adopted in about 95% of prestressed concrete bridge projects while complete prestressing is adopted in only 5% of them. With the objective of covering

Table 3 – Characteristic compressive strength of concrete (reinforced concrete)

Period	Until 1977	1978 to 1983	1984 to 1986	1987 to 2002	Starting from 2003
Class	36	36	45	45	45
f_{ck} (MPa)	18	18	20	20	25

Table 4 – Load factors for serviceability limit state

Period	Until 1977	1978 to 1983	1984 to 1986	1987 to 2002	Starting from 2003
Class	36	36	45	45	45
γ_g	1.00	1.00	1.00	-	-
γ_q	1.20	1.20	1.00	-	-
Ψ_1	-	-	-	0.80	0.50
Ψ_2	-	-	-	0.40	0.30

Ψ_1 : load factor for high frequency combination | Ψ_2 : load factor for almost permanent combination

a vast range of cases while reducing the number of analysis, prestressed concrete bridges are considered with limited prestressing in the present paper. Besides, the prestressed concrete is considered with post-tension and subsequent bond. As usual for prestressed concrete, the design is based on the serviceability limit states (SLS) with the subsequent verification of the ultimate limit state (ULS). The periods and the respective load factors for the serviceability limit state are given in Table 4. The verification of the ultimate limit state uses the same load factors as for reinforced concrete bridges (Table 2). The characteristic strengths of concrete adopted for the prestressed structures, usually higher than that of reinforced concrete, are given in Table 5. To define the characteristic strength, it was assumed that the slab and box bridges are cast-in-place, while the 5 girder bridges are formed by precast beams fabricated at the construction site. In the last case, a lower strength of the concrete deck is considered.

2.3 Nominal bending moment resistance

The nominal strength is calculated using the characteristic values of the concrete and steel properties indicated by NBR 6118 (ABNT [7]). Firstly, the cross section is divided into a series of rectangular strips. Constant deformation and stress in each strip is considered. The position of the neutral axis is determined through iterations until the balance of forces is obtained.

Among the 5 intervals of time set previously, just one of them

considered critical (lower bending moment resistance) for each class is considered in the calculations. It should be pointed out that, for the class 45 bridges, the period between 1984 and 1986 was excluded of the analysis because of its low probability of occurrence compared to others.

2.4 Implementation of the Monte Carlo's technique

The strength of structural elements is considered a random variable due to uncertainties in the materials properties, dimensions and methods of analysis. The evaluation of their statistical parameters, such as the mean value, coefficient of variation, probability density function and cumulative distribution function, requires a great number of materials and test elements, hence resulting in high costs. Alternatively, the behavior of the structural components may be determined through simulations. Particularly, this paper makes use of the Monte Carlo's technique.

2.4.1 Statistical parameters of the materials properties and dimensions

The basic variables are the compressive strength of concrete, the properties of steel and the cross section dimensions. The statistical parameters to be used in the simulations, except for the concrete strength, were obtained from the available literature (Ferreira [5]).

For older bridges, those built up to 1986, the normal distribu-

Table 5 – Characteristic compressive strength of concrete in MPa (prestressed concrete)

Period	Until 1977	1978 to 1983	1984 to 1986	1987 to 2002	Starting from 2003
Class	36	36	45	45	45
Slab					
5 girders	25	25	30	30	35
Box section					
5 girders (deck)	18	18	20	20	25

tion prescribed by NB 1 (ABNT [8]) is used to represent the concrete strength:

$$f_{c28} = f_{ck} + 1.65 \cdot \sigma_d \quad (2)$$

where

σ_d : standard deviation of concrete, equal to 4 MPa for severe control.

With the objective of investigating the characteristics of the concrete produced recently that may be applied to bridges constructed starting from 2003 and as an approach to bridges of the previous period (1987 to 2002), a data collection on the compressive strength of concrete from cylindrical specimens was started. The obtained data refer to ready-mix concretes used at several sites and in a large number of constructions.

The data are plotted on probability paper that aid in choosing the type of distribution and in determining the most important statistical parameters. The abscissa represents the basic variable (in this case, the compressive strength of concrete) while the ordinate represents the cumulative probability. The basic idea is to redefine the vertical scale in such a way that any series of data described by the same type of distribution on the paper is represented by a straight line. The slope of the resultant straight line and its intercept with the horizontal axis supply the mean value and the standard deviation. More details can be found in Nowak and Collins [9].

Considering that the compressive strength of concrete usually follows a normal or lognormal distribution, the two probability papers were tested for each set of data. Figure 1 shows the data of concretes with characteristic strength equal to 20 MPa plotted on a normal probability paper. The relationship between the mean value and the nominal value (characteristic), called bias, and the coefficient of variation (CV) are obtained by ad-

justing a straight line to the lower tail of the distribution, since it is the most important region regarding the reliability aspect.

The results obtained indicated that the lognormal distribution is more appropriate for concretes of characteristic strength equal to 15 MPa and 18 MPa while for concretes of characteristic strength of 20 MPa and above, the normal distribution is recommended (Ferreira [5]).

A summary of the statistical parameters of materials and dimensions used in this paper is presented in Table 6.

2.4.2 Mean bending moment resistance

From the idealized probability distributions for each basic variable, the statistical resistance parameters (mean value and coefficient of variation) of each cross section considered are obtained for 20,000 simulations of Monte Carlo's technique.

3. Live load

A live load model is developed based on trucks weightings carried out by Centrovias Sistemas Rodoviários S/A. The data analyzed was that ranging from January 2001 to October 2002, totaling 184,603 weightings, 126.389 of which were during the first year and 58.214 in 2002.

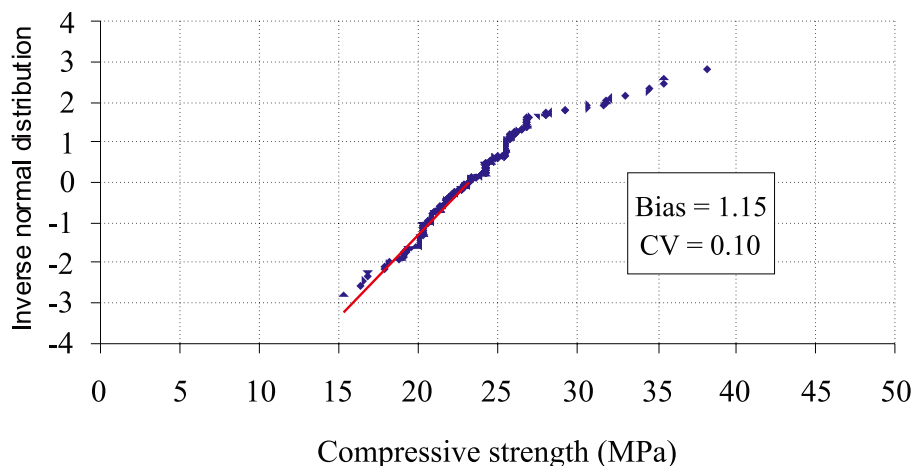
3.1 Selected trucks

In developing the live load model, only those trucks that appear with larger weight and in a larger number on the spreadsheets of weighting from Centrovias were considered based on a procedure already adopted by other authors (Hwang and Koh [10]). Initially, the following vehicles were selected: 2S3e, 2S3, 3S3, 2S2 and 3S2B2. The nomenclature adopted is that suggested by Widmer [11].

3.2 Distance between axles

The weighting spreadsheets, in spite of informing the configuration of each truck, do not indicate the distance between axles. It is known

Figure 1 - Normal probability paper - $f_{ck} = 20$ MPa



that even inside a certain classification, the distance between axles varies from vehicle to vehicle, sometimes significantly.

There is no knowledge on data regarding the distribution of the distance between axles obtained from field observations. Alternatively, this paper uses typical and representative distances determined from the virtual catalog of the company Randon S/A, from information supplied by Transervice Consultoria em Transportes S/C Ltda and also from pictures.

The vehicles and the distances adopted between axles are shown in Figure 2.

3.3 Extrapolation of maximum gross weights

Considering the limited period during which the vehicles were

weighed, the maximum gross weights for each truck should be extrapolated according to the design life of the structures.

Initially, the results of the weighting for each truck are plotted on a normal probability paper. Although the distributions do not follow a normal distribution strictly, this procedure is quite convenient because it permits a general view of the data and indicates tendencies for longer periods than those effectively measured. Besides, it is the method used by other researchers for the development of live load models, among them, Hwang and Koh [10] and Nowak [12].

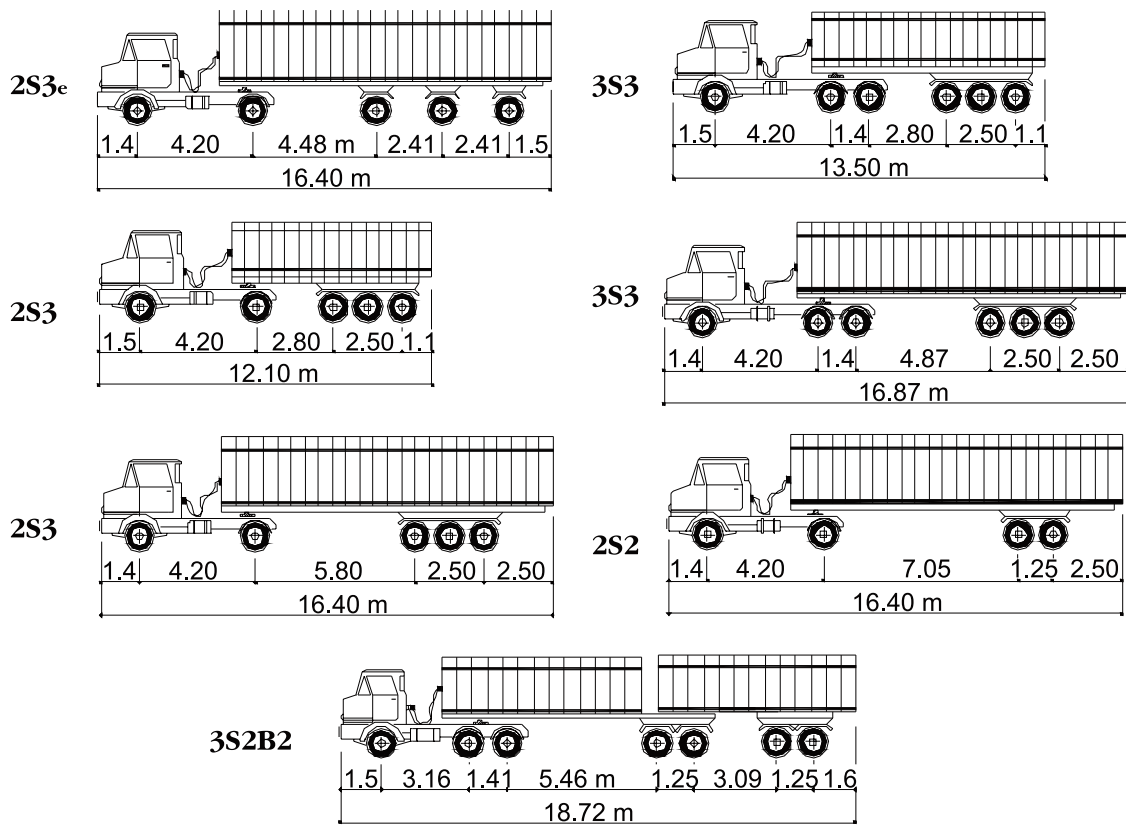
The next stage involves estimating the total number of trucks corresponding to the daily traffic in the area where the weightings were obtained and determining the design life of the structure, assumed in this study equal to be 50 years. From the percentage of each configuration,

Table 6 – Summary of the statistical parameters for simulation

Variable	Notation	Mean value	CV	Distribution Type	
Thickness of flange	h_f (cm)	h_f	$0.5/h_f$	Normal	
Height of section	h (cm)	h	$0.5/h$	Normal	
Width of the section	b_f	b_f	-	-	
Width of web	b_w	$1.01 \times b_w$	0.04	Normal	
Depth from concrete extreme compressive fiber to centroid of steel	d (reinforcing steel)	$0.99 \times d$	0.04	Normal	
	d (prestressing steel)	$1.00 \times d$	0.025	Normal	
Area of reinforcing steel	A_s	A_s	0.015	Normal	
Area of prestressing steel	A_p	A_p	0.0125	Normal	
Compressive strength of concrete after 28 days	f_c (MPa)	$f_{ck} = 18^*$	24.6	0.163	Normal
		$f_{ck} = 25^*$	31.6	0.127	Normal
		$f_{ck} = 20^{**}$	23.0	0.103	Normal
		$f_{ck} = 25^{**}$	28.0	0.093	Normal
		$f_{ck} = 30^{**}$	32.7	0.089	Normal
		$f_{ck} = 35^{**}$	37.5	0.088	Normal
Yield strength of reinforcing steel	f_y (kN/cm ²)	56	0.054	Lognormal	
Ultimate strength of prestressing steel	f_{pu} (kN/cm ²)	$1.03 \times f_{pu}$	0.022	Lognormal	
Modulus of elasticity of reinforcing steel	E_s	21000	0.06	Lognormal	
Modulus of elasticity of prestressing steel	E_p	20000	-	-	

* Until 1983 | ** Starting from 1987

Figure 2 – Schematic representation of the selected LCVs



the total number of occurrences from a certain vehicle in 50 years (n) is obtained. With $z = \Phi^{-1}(1/n)$, where $1/n$ is the level of probability corresponding to n and Φ^{-1} is the inverse of the cumulative distribution function of the standard normal variable, the value that the extrapolation should reach can be determined. The idea is that if all trucks were sampled during 50 years, the heaviest truck would be at the level z . The extrapolation is accomplished through a linear regression that includes the 100 heaviest trucks of each selected vehicle. Figure 3 presents an example for the configuration 3S3 of 13.50 m length. In Table 7, the maximum gross weights obtained from the adopted methodology are shown.

3.4 Critical load cases

The critical scenario for bridge safety analysis may result from the crossing of a single truck or the simultaneous presence of two or more vehicles.

3.4.1 Single truck

In this case, it is admitted that the presence of only one truck over the bridge with weight corresponding to a design life of 50 years is critical for the calculation of the bending moments under real traffic. The simulations carried out indicated that although the heaviest

truck is the 2S2 (see Table 7), the short configuration of the vehicle 3S3 (13.50 m length) is more critical regarding load effects. Because of this, the 3S3 vehicle was tested separately to obtain maximum effects in all selected structures.

It was considered that the maximum gross weight during 50 years of design life is the mean value of a new distribution, according to Nowak [12].

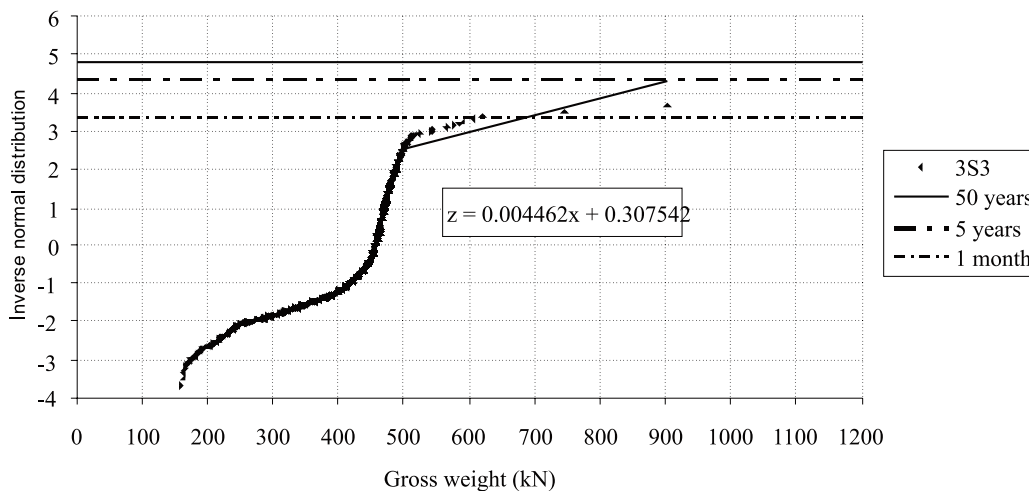
3.4.2 Simultaneous presence of two trucks

The analysis of 2 trucks over the bridges requires the determination of the relative position and the relation between their weights. Considering that the probability of 2 trucks passing over the bridge sufficiently close and in relative positions that cause a maximum bending moment situation is smaller than for an isolated truck (smaller number of occurrences), their gross weight is also expected to be lower.

Statistical data that supply the probabilities of simultaneous occurrences on bridges are not available in Brazil. Therefore, based on Hwang and Koh [10] and Nowak [12] as references, the probabilities of simultaneous presence adopted for this study are given in Table 8.

According to Table 8, the two cases to be analyzed are 2 trucks with probability of 1/500 (hypothesis 2) and 2 trucks with different prob-

Figure 3 – Extrapolation for the truck 3S3 of 13.50 m length



abilities (hypothesis 1), one of them with probability varying between 1/75 and 1/50, and the other of mean weight. In the case of trucks on the same lane, several distances between them were verified, highlighting that the dynamic portion of the live load effect is not considered for distances between trucks of below 15 meters.

In the present study, the main truck is assumed to be a 3S3 of 13.50 m length. Assuming a simultaneous presence, it is considered that the second truck is a 2S3 of 12.10 m length since the total number of 2S3 trucks represents approximately 30% of the total observed trucks.

It is worth pointing out that the case of vehicles positioned side by side refers to trucks on adjacent lanes and in any relative positions such that the resulting bending moment observed is maximum.

3.4.3 Simultaneous presence of 3 or more trucks

The literature consulted does not provide any indications regarding the probability of 3 or more trucks on the bridge, which is consid-

ered low and of difficult investigation. Besides, the position of a third vehicle will match smaller values of the influence line. Notwithstanding, the 3-truck case is pertinent only in long bridges and/or bridges with several traffic lanes. In case of long bridges, the dead load prevails and the effect of live load becomes less important while in short bridges and bridges with a smaller number of lanes, there is not even enough space for the positioning of 3 trucks. Thus, it is considered that the case of 3 or more trucks is not indispensable for the live load analysis and the safety of the bridges treated here may be reliably measured from the 1 or 2-truck analysis only.

3.5 Maximum bending moments

The proposed live load model was analyzed in all selected typical bridges. It was noticed that, generally, the case of 2 trucks positioned side by side defines the critical condition for wide deck bridges while for narrow deck bridges, the 1 isolated vehicle or 2 trucks positioned side by side are most often the critical situation. The mean dynamic load used in the analyses is that suggested by Nowak [12]. In the case of 1 isolated truck, the dynamic portion of

Table 7 – Estimate of maximum gross weights in 50 years (kN)

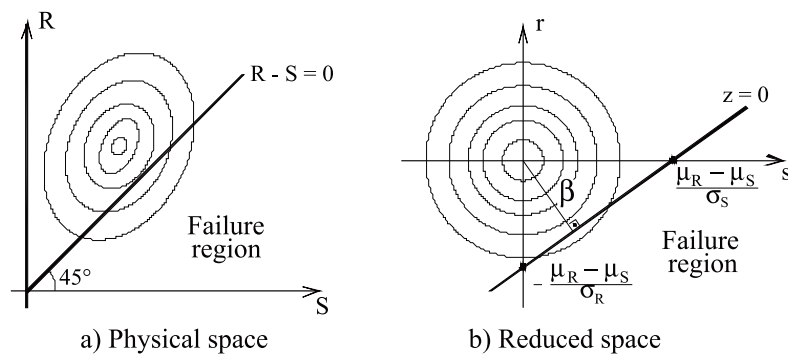
Truck	Gross weight in 50 years
2S3 _e	521.8
2S3 of 12.10 m	696.9
2S3 of 16.40 m	723.9
3S3 of 13.50 m	1009.1
3S3 of 16.87 m	1069.6
2S2	1081.6
3S2B2	664.1

Table 8 – Adopted probabilities of multiple presence

Hypothesis	Trucks in line		Trucks side by side	
	ND	WD	ND	WD
1	1/75	1/75	1/50	1/75
2	1/500	1/500	1/500	1/500

ND: Narrow deck (bridges with deck width equal to 8 meters)
WD: Wide deck (bridges with deck width equal to 12 meters)

Figure 4 - Reliability index definition



the bending moment is considered equal to 15% of the static bending moment and, in the case of 2 trucks, the recommended value is 10%. The coefficient of variation in both cases is 0.80.

Taking into consideration all the cases studied, observation showed that for the class 36 bridges, the relationship between the mean value resulting from the proposed live load model and the value obtained with the normative load model varies between 0.69 and 1.80. In class 45, these values were shown to vary between 0.49 and 1.42. The large variability is justified by the diversity of structural systems, longitudinal arrangements, spans and deck widths. It must be pointed out that, due to the characteristics of the Brazilian code to bridge loads, this relationship tends to decrease with increase span length.

4. Reliability analysis

4.1 Introduction

According to Nowak and Collins [9], the reliability of a structure is its ability to meet design requirements during the design life or the probability that the structure will not fail in carrying out its functions.

Mathematically, the limit states are represented by a function called limit state function. Regarding the ultimate limit state, the limit state function associates to the concept of safety margin:

$$Z = R - S \quad (3)$$

where:

Z: safety margin;

R: capacity or resistance;

S: internal effects, demand or total load effect.

In the case of bridges,

$$S = D + L + E + \dots \quad (4)$$

where:

D: dead load effect;

L: live load effect;

E: environment load effect (wind, temperature, earthquake etc).

The boundary between the desired and undesired performance is given by $Z = 0$. Hence if $Z > 0$, the structure is said to be safe while for $Z < 0$ the structure is said to be unsafe. Therefore, the probability of structural failure is given by the probability that the resistance R is smaller than the effect of the applied load S, as follows:

$$P_f = P[R \leq S] = P[Z \leq 0] = \int_{-\infty}^0 f_Z(z) dz = F_Z(0) \quad (5)$$

where:

$f_Z(z)$: probability density function of the variable Z;

$F_Z(0)$: value of the cumulative distribution function at $Z = 0$.

In terms of R and S, equation [5] may be also represented by:

$$P_f = \int_{-\infty}^{+\infty} F_R(x) f_S(x) dx \quad (6)$$

where:

$f_S(x)$: probability density function of the variable S;

$F_R(x)$: cumulative distribution function of the variable R.

However, the large number of random variables that may be involved in many practical problems turns the integration of equation [6] almost always unfeasible. Alternatively, the structural safety is measured in terms of the reliability index β , defined as the shortest distance from the origin of the reduced variables space to the failure surface (Figure 4). In the case of only two variables, the failure surface is a line corresponding to $z(r,s) = 0$, where:

$$r = \frac{R - \mu_R}{\sigma_R} \quad (7)$$

$$S = \frac{S - \mu_S}{\sigma_S} \tag{8}$$

If R and S follow independent normal probability distributions, the reliability index links directly with the failure probability in the form:

$$P_f = \Phi \left[-\frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \right] = \Phi \left[-\frac{\mu_Z}{\sigma_Z} \right] = \Phi[-\beta] \tag{9}$$

and

$$\beta = \frac{\mu_Z}{\sigma_Z} \tag{10}$$

where:

Φ : cumulative distribution function of the standard normal variable;

μ : mean value of the variables;

σ : standard deviation of the variables.

Figure 5 shows the failure probability, represented by the shaded area, and the reliability index β , that supplies the number of standard deviations that the mean value of the safety margin is away from the limit state. If the involved variables do not follow a normal distribution, the relationship between β and the failure probability is only approximate.

More details may be seen in Nowak and Collins [9].

4.2 Methodology

The developed reliability analysis has 2 functions:

- a) To determine the reliability indexes of the most critical cross sections submitted to positive and negative bending moments of each bridge when subjected to the proposed live load model;
- b) Once the target reliability index is fixed, to supply the bending moment due to live load so that the established safety level is not exceeded.

In the reliability analysis, it is admitted that the total load effect

corresponding to the sum of the effects from several actions is a normal variable while the resistance corresponding to the product of random variables is considered a lognormal variable. Hence the Central Limit Theorem (Nowak and Collins [9]) is applied. This theorem says that the sum of independent random variables tends to a normal distribution as the number of variables increases if none of them is dominant.

Although in this study the number of actions is relatively small to satisfy the Central Limit Theorem, the methodology of Nowak [12] is maintained with the objective of comparing the results and fixing the target reliability index.

The Rackwitz-Fiessler procedure is used (Nowak and Collins [9]) and the limit state function is $Z = R - S$, where R is the cross section resistance to bending and S is the total load effect (self-weight of the structure + self-weight of the asphaltic covering + live load, including the dynamic portion).

The results of the reliability analysis presented in next paragraphs may be seen with more details in Ferreira [5].

4.3 Reinforced concrete

The observed results demonstrate that bridges of class 36 present lower reliability indexes when compared to bridges of class 45. That difference is more pronounced in shorter spans, because the dead load to live load relationship is smaller. It is worth pointing out that for a same structure, bridges of classes 36 and 45 were designed considering the same self-weight.

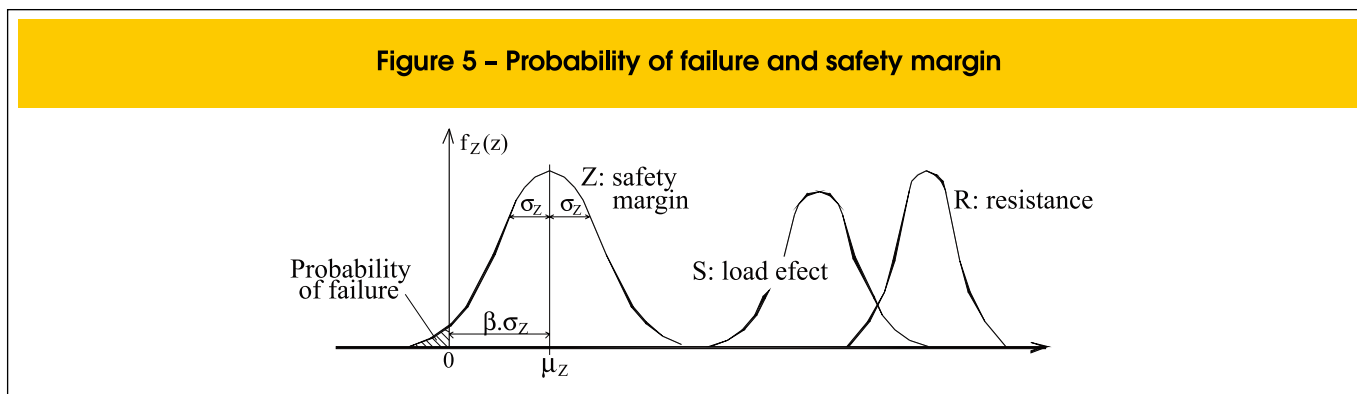
In terms of positive bending moment, within a given class and structural system, shorter span bridges were shown to generally exhibit lower reliability indexes and therefore lower safety levels. This conclusion was also obtained by Ghosn [4].

The negative bending moments in cantilever parts present very low reliability indexes. The reliability indexes between similar bridges of narrow deck and wide deck depend on the structural system and on the span. In the case of 5 girder narrow deck bridges, the safety is found to be larger in bridges with crossbeams compared to bridges without crossbeams while in wide deck bridges, the reverse is true.

4.4 Prestressed concrete

4.4.1 Ultimate limit state

Usually, reliability indexes obtained for prestressed concrete bridges considering the ultimate limit state are found to be larger than



the values obtained for reinforced concrete structures. This fact is attributed to differences in the preliminary design, to the calculation procedure of cross sections and also to the lower variability of the statistical parameters of materials properties and dimensions.

Bridges with wide deck tend to exhibit larger levels of safety compared to narrow deck bridges. In the case of 5 girder bridges, those ones with crossbeams were shown to present larger reliability indexes compared to structures without crossbeams.

Besides, like in reinforced concrete bridges, the reliability indexes are shown to increase with the increase of the span. However, in slab bridges and in 5 girder bridges, the reliability indexes are observed to be larger for class 36 due to differences in the hypotheses considered when verifying the serviceability limit states.

4.4.2 Serviceability limit states

The cracking limit state presents lower reliability indexes than the ultimate limit state. It is also emphasized that class 36 bridges present larger reliability indexes compared to bridges of class 45 due to the different load factors for serviceability.

Following the tendency that has already taken place in the case of reinforced concrete and for the ultimate limit state in the case of prestressed concrete, slab bridges present larger reliability indexes compared to similar bridges of other structural systems. Here also, the favorable effect of crossbeams in 5 girder bridges is again verified.

5. Bridge formula calculation

The basic efficiency requirements that the proposed equations should look for are:

- all vehicles (or groups of axles) that cause lower reliability indexes than the target safety level should have their weights restricted;
- the equation should not limit the weight of vehicles (or group of axles) beyond the limit imposed from the target reliability index;
- the equation must be simple and easy to use.

5.1 Calculation procedure

The steps necessary to obtain the bridge formulas and which will be applied to each bridge of the selected data base are the following ones:

- assume that a group of axles of weight W and length B satisfies the equation;
- adopt a distribution for the weight W along the length B so that the bending moments obtained for real trucks and for the assumed distribution are approximately the same. In this research, the real trucks are represented by 11 uniformly distributed axles of the same load;
- vary B between 2.5 and 30 meters;
- for each B , calculate the bending moment (M_r) equivalent to a unit load uniformly distributed along its length, being

$$\text{attributed to each axle } \frac{1 \cdot B}{1};$$

- obtain the bending moment that corresponds to the target safety level (β_t) and subtract the effects of the dead load

(self-weight of the structure and self-weight of the asphaltic covering) and of the dynamic portion of the live load.

Considering that the truck of the bridge formula may not be isolated when moving over the bridges, also subtract the bending moment corresponding to the accompanying truck (secondary vehicle). The resulted bending moment is called M_{max} and the adopted secondary vehicle is a 2S3 of 12.1 m length with probability of 1/500 (gross weight equal approximately to 560 kN);

- calculate the weight limit from: $W = \frac{M_{max}}{M_r} \cdot B$;

- after carrying out the above stages for several values of B , obtain the curves W as a function of B . Repeating the procedure for all bridge types of the data base, it is obtained the envelope that the gross weights should satisfy so that a reliability index larger or equal to beta target is guaranteed;
- find an algebraic expression that represents, in a simple way, the envelope previously obtained.

5.2 Adoption of the target reliability index (β_t)

The determination of the target reliability index is based on the performance of existent bridges subjected to the proposed live load model and on recommendations in technical literatures. In agreement to Ghosn [4] and Nowak [12], only one value of β is used, independently of the structural system, class (36 or 45), length and number of lanes. Also, no differentiation is made between positive and negative bending moment. Only the concept of redundancy and the type of the state limit are taken into consideration when fixing β_t .

For the ultimate limit state, the value of β_t used in the calculation of the bridge formula for 2 girder bridges and 5 girder bridges without crossbeams considering its low load redistribution is 4. In other structural systems, beta target is assumed to be 3.5. The probabilities of failure associated to those reliability indexes are $3.17 \cdot 10^{-5}$ and $2.33 \cdot 10^{-4}$, respectively.

Considering that for prestressed bridges, the failure to meet the state limit of cracking presents less severe consequences to the structures compared to the ultimate limit state, and assuming that the environment is not aggressive, a value of 0.5 is adopted for β_t (probability of failure corresponding to 0.31).

6. Proposed equations

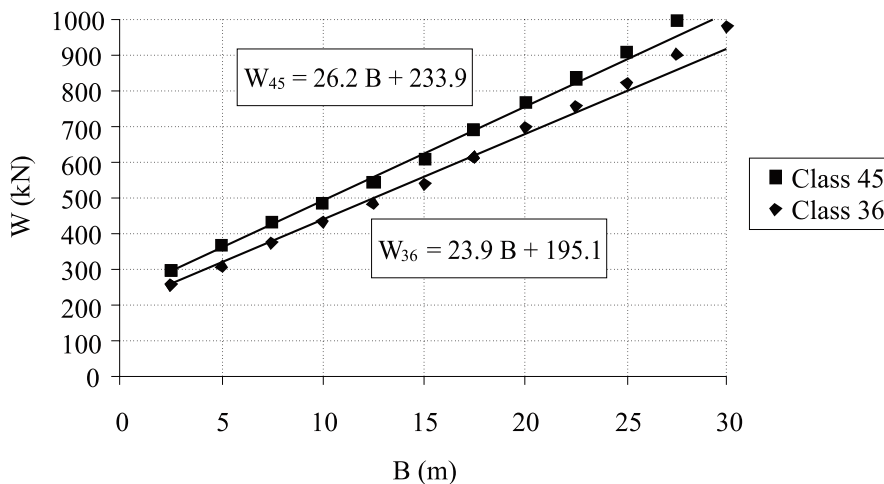
Two equations are supplied for each bridge class, one equation being for the ultimate limit state and other for the serviceability limit state. While the first must be obeyed in a more rigorous way, the application of the second is attributed to an engineer's decision.

6.1 Ultimate limit state (ULS)

The proposed equations for the ultimate limit state are given in Figure 6 and the result of the equations in table form is shown in Table 9.

The situation of some typical trucks regarding the ultimate limit state is verified in Table 10. Besides the distance between extreme axles (total gross weight of the vehicle), groups of intermediate axles are also tested in some cases. When the nominal weight is larger than that obtained with the corresponding bridge formula,

Figure 6 - Proposed bridge formula for the ultimate limit state



the group of axles does not respect the limits obtained in this study and therefore should have its weight restricted.

The analysis of Table 10 indicates the following restrictions:

- a) 3S3: without restrictions as far as it respects the legislation;
- b) 3S2B2: does not satisfy the bridge formula for class 36, especially in relation to the 4 rear axles. However, the difference is not significant;
- c) 3S3B3: the 6 rear axles of the configuration used in the example do not respect the bridge formula for class 36. Regarding the total gross weight of the vehicle, the difference

- is not significant;
- d) 3S2A2S2 of 19.80 meters length: does not satisfy the weight limits imposed for class 36. For class 45, the maximum total gross weight is not satisfied;
- e) 3S2A2S2 of 25 meters length: considering the total weight, it may be admitted that there are no restrictions. However, it presents a group of axles in the same conditions of the 3S2B2 vehicle that is slightly above the weight limits for class 36.

6.2 Serviceability limit state (SLS)

Figure 7 shows the weight limits considering the serviceability limit state (cracking) for prestressed concrete bridges. The same results are shown in Table 11.

The effect of the trucks treated in the ultimate limit state, but this time considering the serviceability limit state, is in Table 12.

The results show that:

- a) class 36: the only restriction presented is in relation to truck 3S2A2S2 of 19.8 meters;
- b) class 45: presents restrictions for the circulation of all trucks, especially in what refers to the vehicle 3S3B3 and 3S2A2S2 of 19.8 meters length.

It is pointed out that other configurations of the same trucks can make possible their use for the respective states limits.

7. Conclusions and final considerations

Based on the developed study the following conclusions are obtained:

- a) considering the ultimate limit state, the weight limits for any groups of consecutive axles of loaded vehicles can be supplied by the equations:

$$\text{Class 36: } W_{36} = 23.9 \cdot B + 195.1$$

(11a)

Table 9 - Gross weights obtained from the proposed bridge formula for ULS

B (m)	W (kN)	
	Class 36	Class 45
2.5	255	299
5.0	315	365
7.5	374	430
10.0	434	496
12.5	494	561
15.0	554	627
17.5	613	692
20.0	673	758
22.5	733	823
25.0	793	889
27.5	852	954
30.0	912	1020

Table 10 – Situation of some vehicles according to ULS

Truck	Gross weight (kN)	Length (m)	B (m)	Nominal weight (kN)	W (kN)	
					Class 36	Class 45
3S3	450	13.50	10.90	450	456	519
3S2B2	570	18.72	5.59	340	329	380
			15.62	570	568	643
3S3B3	740	24.90	11.15	510	462	526
			22.49	740	733	823
3S2A2S2	740	19.80	4.65	340	306	356
			17.53	740	614	693
3S2A2S2	740	25.00	5.55	340	328	379
			22.73	740	738	829

Class 45: $W_{45} = 26.2 \cdot B + 233.9$

(11b)

W: maximum gross weight in kN on any group of two or more consecutive axles;

B: length of the axle group in meters.

b) the use of the weight limits established above indicates restrictions to the circulation of some vehicles, especially in class 36 bridges. The results also show that, among the verified vehicles, the 3S2A2S2 truck of 19.8 meters length and 740 kN is the most incompatible to the studied Brazilian typical bridges;

c) the developed equations for the serviceability limit state are:

Class 36: $W_{36} = 20.2 \cdot B + 319.1$

(12a)

Class 45: $W_{45} = 20.1 \cdot B + 213.6$

(12b)

d) in this last case, the proposed equations for prestressed

Figure 7 – Proposed bridge formula for the serviceability limit state

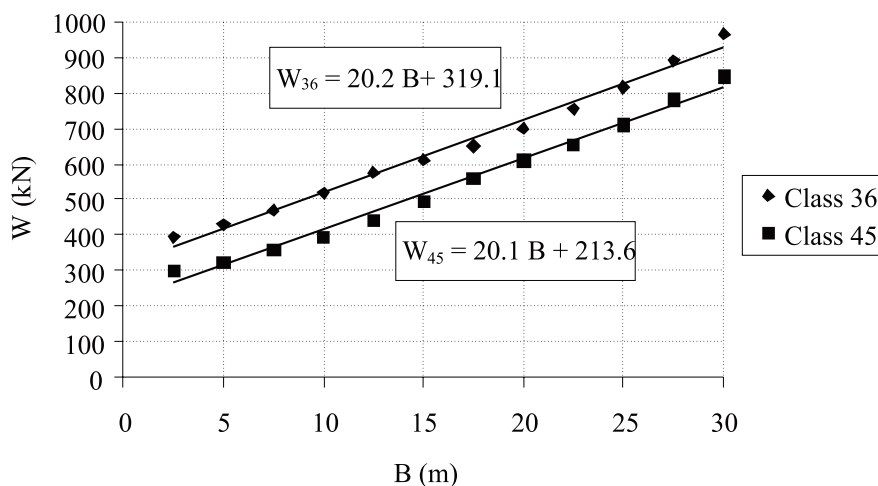


Table 11 – Gross weights obtained from the proposed bridge formula for SLS

B (m)	W (kN)	
	Class 36	Class 45
2.5	370	264
5.0	420	314
7.5	471	364
10.0	521	415
12.5	572	465
15.0	622	515
17.5	673	565
20.0	723	616
22.5	774	666
25.0	824	716
27.5	875	766
30.0	925	817

structures result in low gross weights for bridges of class 45 indicating that the current load factors in service reduce the safety regarding the limit state of cracking compared to previous periods.

It should be reminded that the study analyzed the most common structural systems found at Sao Paulo Department of Transportation highways and also to attend to most usual design situations. Spans up to 40 meters were considered respecting the particularities of each structural system.

Within the premises and hypotheses assumed in this work, it may be emphasized that: a) the bending moment in the superstructure of bridges classes 36 and 45 is considered; b) the bridges are assumed to be straight, with no skew and with even supports; c) the dead load and the vertical effect of the live load is verified; d) the study does not include fatigue analysis in concrete bridges; e) the proposed bridge formulas do not contemplate cantilever parts with less than 5 meters; f) the maximum gross weights were reduced in 5%, considering tolerance allowed by the Brazilian legislation; g) it is assumed that the bridges have been appropriately designed and executed and besides, that they do not present any pathologies that jeopardize their capacity and; h) the axles weight limits allowed by Brazilian legislation must be respected.

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Table 12 – Situation of some vehicles according to SLS

Truck	Gross weight (kN)	Length (m)	B (m)	Nominal weight (kN)	W (kN)	
					Class 36	Class 45
3S3	450	13.50	10.90	450	539	433
3S2B2	570	18.72	5.59	340	432	326
			15.62	570	635	528
3S3B3	740	24.90	11.15	510	544	438
			22.49	740	773	666
3S2A2S2	740	19.80	4.65	340	413	307
			17.53	740	673	566
3S2A2S2	740	25.00	5.55	340	431	325
			22.73	740	778	670

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