Reliability-based calibration of Brazilian structural design codes used in the design of concrete structures

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Abstract

This paper presents a reliability-based calibration of partial safety factors for Brazilian codes used in the design of concrete structures. The work is based on reliability theory, which allows an explicit representation of the uncertainties involved in terms of resistances and loads. Regarding the resistances, this study considers beams with concrete of five classes (C20, C30, C40, C50 and C60), three ratios between base and effective depth (0.25, 0.50 and 0.75), three longitudinal reinforcement ratios ($\rho_{\min}$, 0.5% and $\rho_{\max}$) and three transverse reinforcement ratios ($\frac{A_s}{s}$ min, $\frac{A_s}{s}$ and $\frac{A_s}{s}$ max).

In terms of loads, this work considers seven ratios between live loads and permanent loads ($\frac{q_n}{g_n}$), and seven ratios between wind loads and permanent loads ($\frac{w_n}{g_n}$). The study also adopts a single value for the target reliability index ($\beta_{\text{target}} = 3.0$). Results show that the optimized set of partial safety factors leads to more uniform reliability for different design situations and load combinations.

Keywords: code calibration, concrete structures, structural safety, reliability, safety, NBR 8681, NBR 6118.

Resumo

Este artigo apresenta a calibração, baseada em confiabilidade, dos coeficientes parciais de segurança das normas brasileiras utilizadas no dimensionamento de estruturas de concreto. O trabalho está fundamentado na teoria de confiabilidade, que permite uma representação explícita das incertezas envolvidas em termos das resistências e ações. No que tange às resistências, são consideradas vigas projetadas para resistir a esforços de flexão e cisalhamento com concretos de cinco classes (C20, C30, C40, C50 e C60), três razões entre base e altura útil (0.25, 0.50 e 0.75), três taxas geométricas de armaduras longitudinais ($\rho_{\min}$, 0.5% e $\rho_{\max}$) e três taxas de armaduras transversais ($\frac{A_s}{s}$ min, $\frac{A_s}{s}$ e $\frac{A_s}{s}$ max).

No tocante às ações, são consideradas sete razões entre os carregamentos acidental e permanente ($\frac{q_n}{g_n}$), e sete razões entre os carregamentos de vento e permanente ($\frac{w_n}{g_n}$). O estudo ainda adotou um único valor para o índice de confiabilidade alvo ($\beta_{\text{target}} = 3.0$). Os resultados mostraram que os conjuntos otimizados dos coeficientes parciais de segurança conduzem a uma confiabilidade mais uniforme para diferentes situações de projeto e combinações de carregamentos.

Palavras-chave: calibração de norma, estruturas de concreto, confiabilidade, segurança, NBR 6118, NBR 8681.
1. Introduction

Every structural design has uncertainties associated to construction materials and to the actions it will be subject to during execution and the useful life. Brazilian structural codes incorporate these uncertainties through limit state design, a method that establishes boundaries between desirable and undesirable structural behaviors. According to this method, the safety of a structure is determined by its ability to support actions without reaching any ultimate limit state or serviceability limit state. The ultimate limit states are associated to loss of equilibrium of the structure or parts of it; while the serviceability limit states are related to the functionality of the structure. Limit state design involves use of partial safety factors that control the risk against failure of a structure. Therefore, there are factors to reduce the resistance of the structural elements and to increase the actions, creating a margin of safety in relation to the main sources of uncertainties.

A reliability-based calibration process converted the American structural codes to this format. However, there is no clear evidence in the literature that the partial safety factors indicated in European codes derived from a generalized calibration process. The safety factors indicated in Brazilian codes were not properly calibrated and were adapted from American and European codes. It is imperative that national codes be calibrated based on uncertainties that reflect the Brazilian reality, in terms of materials, actions and calculation models.

2. Objectives

This work aims to present a first study about the partial safety coefficients indicated in the main Brazilian codes used in the design of concrete structures: structural actions code [1] and concrete structures design code [2]. The study is based on structural reliability theory, which allows an explicit representation of uncertainties through consideration of resistances and loads as random variables, resulting in a quantitative estimation of safety: the reliability index ($\beta$).

The work involves a calibration methodology oriented to obtain a set of partial safety factors that minimizes the variations of the reliability indexes of different types of reinforced concrete beams projected according to Brazilian codes, in relation to the chosen target reliability index ($\beta_{target}$). This study is important because it presents sets of partial safety factors that best reflect the reality of Brazilian concrete structures. It also reveals the need for an extensive calibration that contemplates other types of structural materials and elements.

3. Context

The code calibration process is intended to adjust safety factors that lead to designs with a desired level of reliability. In this way, calibration is the process of finding the set ($\gamma_s$, $\gamma_g$, $\gamma_q$, $\gamma_w$, $\psi_z$ and $\psi_d$) that minimizes the variations of the reliability indexes of the most diverse structures designed within the scope of a certain code, with respect to the target reliability index.

In the 1970s, the first publications were made with statistical results of loads, materials strength and load combinations for different types of structures [5]. These results allowed a first reliability-based calibration of American structural design codes in the 1980s. There is no clear evidence that European codes have gone through a similar calibration process, despite the clear effort of the Joint Committee on Structural Safety (JCSS), composed by professionals from different countries and involved with structural reliability research. Since the beginning of this century, research has been conducted with respect to American concrete structures [6-10].

As the Brazilian structural design codes have been adapted to the limit state format from the American and European codes, it is evident that Brazilian partial safety factors have not been calibrated for the reality of the country. A first approach to the calibration of Brazilian codes was made by Souza Jr [11], dealing specifically with steel structures. Subsequently, Nova and Silva [12] produced a preliminary calibration of Brazilian codes used in the design of prestressed concrete bridges.

4. Methodology

The present work deals with reliability-based calibration of the partial safety factors indicated in the main Brazilian codes used in the design of concrete structures. The procedure follows the main guidelines in Melchers and Beck [13]. Briefly, the calibration involves two major steps: the collection of statistics related to the reality of materials, loads and design models in Brazil, and the calibration of partial safety factors. It should also be mentioned that the reliability problem was solved using the STRAND program - Structural Reliability Analysis and Design – developed by Beck [14] at the Department of Structural Engineering of the School of Engineering of São Carlos.

5. Brazilian statistics

In this section we present the random variables related to Brazilian concrete structures. The distributions of the variables not available in the literature were adjusted based on the Chi-Square, Kolmogorov-Smirnov and Anderson-Darling goodness-of-fit tests, after exclusion of outliers.

5.1 Resistance variables

The following resistance variables related to concrete beams built in Brazil were adopted: concrete compressive strength ($f_c$), yield strength of reinforcing bars ($f_y$), cross section base ($b$), effective depth ($d$) and professional factor or resistance model uncertainties ($\mu_{Rm}$). A summary of the results concerning these variables is shown in Table 1; it is important to notice that the means are expressed as a function of their respective characteristic or nominal values.

Statistics in Table 1 were obtained from results of axial compression tests at 28 days performed in more than 39 thousand cylindrical specimens of concrete molded in loco all over Brazil, between 2011 and 2016, as reported by Santiago and Beck [15, 16, 17]. As in the work of Nowak et al. [8], the parameters of the probability distribution functions of the variables were ob-
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Table 1
Resistance random variables

<table>
<thead>
<tr>
<th>Random variable</th>
<th>Distribution</th>
<th>Mean</th>
<th>C.V.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_c$</td>
<td>Normal</td>
<td>$1.30f_{ck}$</td>
<td>0.20</td>
</tr>
<tr>
<td>C20</td>
<td>Normal</td>
<td>$1.22f_{ck}$</td>
<td>0.15</td>
</tr>
<tr>
<td>C30</td>
<td>Normal</td>
<td>$1.16f_{ck}$</td>
<td>0.11</td>
</tr>
<tr>
<td>C40</td>
<td>Normal</td>
<td>$1.11f_{ck}$</td>
<td>0.10</td>
</tr>
<tr>
<td>C50</td>
<td>Normal</td>
<td>$1.10f_{ck}$</td>
<td>0.09</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Normal</td>
<td>$1.22f_{yk}$</td>
<td>0.04</td>
</tr>
<tr>
<td>$b$</td>
<td>Normal</td>
<td>$b_n$</td>
<td>(4+0.006$b_n$)/$b_n$</td>
</tr>
<tr>
<td>$d$</td>
<td>Normal</td>
<td>$d_n$</td>
<td>10 mm/$d_n$</td>
</tr>
<tr>
<td>$E_{ci}$</td>
<td>Flexural resistance</td>
<td>Normal</td>
<td>1.02</td>
</tr>
<tr>
<td>Shear resistance</td>
<td>Normal</td>
<td>1.075</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 2
Quantification of specimens by concrete class

<table>
<thead>
<tr>
<th>Classe</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>C20</td>
<td>4511</td>
</tr>
<tr>
<td>C25</td>
<td>1053</td>
</tr>
<tr>
<td>C30</td>
<td>6685</td>
</tr>
<tr>
<td>C35</td>
<td>7804</td>
</tr>
<tr>
<td>C40</td>
<td>3982</td>
</tr>
<tr>
<td>C45</td>
<td>2527</td>
</tr>
<tr>
<td>C50</td>
<td>13272</td>
</tr>
</tbody>
</table>

Table 3
Quantification of specimens by reinforcing bar size

<table>
<thead>
<tr>
<th>Ø (mm)</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>3352</td>
</tr>
<tr>
<td>12.5</td>
<td>2416</td>
</tr>
<tr>
<td>16</td>
<td>1441</td>
</tr>
<tr>
<td>20</td>
<td>571</td>
</tr>
<tr>
<td>25</td>
<td>961</td>
</tr>
</tbody>
</table>

Figure 1
Recommended parameters for the probability distribution functions of the variable $f_c$. 

Table 2 presents a quantification of the specimens according to the strength class. It should be mentioned that these results were provided by the following companies, educational institutions and laboratories: AJL Engenharia, Centro de Tecnologia da UFAL, CONSULTARE Laboratório, CSP Projetos e Consultoria, EGELETE Engenharia, ITAIPU BINACIONAL, Laboratório de Ensaios de Materiais da FACENS, Laboratório de Materiais de Construção e Técnicas Construtivas da UNIVASF, MPA Controle Tecnológico, SENAI-DF, SILCO Engenharia, TECNOL Tecnologia em Concreto, TECNOCON Engenharia e VENTUSCONE Soluções em Concreto.

The variable $f_y$, on the other hand, was based on results from tensile tests performed in more than 8.7 thousand samples of CA-50 bars with different diameters and produced in several batches in Brazil throughout 2016. Table 3 presents a quantification of the specimens according to the size. It should be noted that these results were provided directly by ArcelorMittal Brasil, which is the largest producer of steel in Latin America, and one of the main manufacturers of reinforcing bars in Brazil.

The impossibility of carrying out an experimental study associated with the scarcity of publications on the subject resulted in...
5.2 Load variables

The following load variables related to beams built in Brazil were adopted: dead load \( (g) \), arbitrary point in time live load \( (q_{apt}) \), 50 year extreme live load \( (q_{50}) \), annual extreme wind load \( (w_1) \), 50 year extreme wind load \( (w_{50}) \) and load model uncertainties \( (\varepsilon_m) \).

A summary of the results concerning these variables is shown in Table 4; it is important to note that the means are expressed as a function of their respective nominal values.

The dead load variable \( g \) was based on results sent by different structural engineers that determined the weight of the same building based on the return given by the construction companies that hired them. The multi-storey residential building represents a generalization of the most commonly constructed buildings in Brazil. The following structural engineers participated in the study: Carlos Baccini, Cesar Pinto, Daniel Miranda, Douglas Couto, Enio Barbosa, Fernando Stucchi, Josafá de Oliveira Filho, Luiz Cabral, Murilo Marques, Paulo Sousa, Rodrigo Nurnberg e Vitor Hugo.

The life load variables \( q_{apt} \) and \( q_{50} \) were based on two reference tributary areas and two shape factors from the stochastic model proposed in JCSS [19]. In this way, the load was divided in two independent parts - sustained and intermittent – in which the time between changes was represented by an exponential distribution and the number of changes by a Poisson process. From the maximum load obtained by the sum between the sustained and intermittent loads in a reference period, it was possible to adjust distributions for both variables based on the revised load values prescribed by the Brazilian code about loads for structural design [21]. Wind load variables \( w_1 \) and \( w_{50} \) assumed the results proposed by Beck and Souza Jr. [4], which were based on wind speed series obtained in Brazilian meteorological stations.

Due to the lack of papers on the uncertainties associated with the definitions of effects in concrete structures design, the random variable \( \varepsilon_m \) was based on the information provided by the JCSS [19].

6. Calibration

Calibration procedure is presented in this section, as well as the cases considered in the determination of the new Brazilian partial safety factors indicated in the structural safety code [1] and in the concrete structures design code [2].

6.1 Procedures

The calibration of the safety factors was formulated as a reliability-based design optimization (RBDO) problem, in which the uncertainties involved, represented as random variables (as shown in Tables 1 and 4), were considered explicitly. The calibration problem can be stated as:

\[
\begin{align*}
\text{minimize: } D_r &= \sum_{i=1}^{m} \sum_{j=1}^{n} \left( \beta_{\text{target}} - \min \left( \beta_{ijk} \right) \right)^2 \cdot w_i \\
\text{subject to: } \beta_{ijk} &= \frac{1}{\Phi^{-1}\left( \frac{\psi_{ij} - \psi_{\text{ref}} \cdot \psi_{w}}{\psi_{\text{ref}}} \right)} - \frac{1}{\Phi^{-1}\left( \frac{\psi_{ij} - \psi_{\text{ref}} \cdot \psi_{w}}{\psi_{\text{ref}}} \right)}
\end{align*}
\]

where \( m \) and \( n \) are the load ratios considered, \( \beta_{\text{ref}} \) is the reliability index calculated for the load ratios \( ij \), \( w_i \) is the weight of each load ratio in the combination, according to the relative importance of that design case (Table 5), and \( k \) is the critical limit state among the cases considered in Eq. 2.

\[
\begin{align*}
g_1 \left( \psi \right) &= \varepsilon_m \cdot R \left( c_i, \psi, b \cdot d \right) - \varepsilon_m \cdot S \left( g + q_{50} + w_1 \right) = 0 \\
g_2 \left( \psi \right) &= \varepsilon_m \cdot R \left( c_i, \psi, b \cdot d \right) - \varepsilon_m \cdot S \left( g + q_{apt} + w_{50} \right) = 0
\end{align*}
\]

These two equations are valid for the five combinations that derive from the combination equation proposed by the structural safety...
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Reliability indexes were evaluated via the First Order Reliability Method (FORM) [22]. FORM is considered adequate in relation to processing speed, given the large number of reliability indexes that were calculated, as well accurate in dealing with low dimensionality limit state equations that do not present large nonlinearities.

The target reliability index was set as \( \beta_{\text{target}} = 3 \), as this corresponds to the mean of reliability indexes obtained before the calibration; this is also a reference number recommended in Melchers and Beck [13].

Finally, the optimization problem was solved through the Particle Swarm Optimization algorithm (PSO), which is a meta-heuristic algorithm oriented at identifying the global minimum in non-convex design spaces [23].

6.2 Structural configurations

This work considered beams with concrete of five classes (C20, C30, C40, C50 and C60), three ratios between base and effective depth (0.25, 0.50 and 0.75), three longitudinal reinforcement ratios (\( \rho_{\text{min}}, 0.5\% \) and \( \rho_{\text{max}} \)) and three transverse reinforcement ratios.

It should be mentioned that \( \rho_{\text{min}} \) is a function of the concrete compressive strength (\( f_{\text{ck}} \)) and \( \rho_{\text{max}} \) is a function of the maximum height of the neutral line in the cross section (\( x/d \leq 0.45 \) for concrete with \( \leq 50 \) MPa and \( x/d \leq 0.35 \) for concrete with \( 50 \) MPa < \( f_{\text{ck}} \) < 90 MPa).

6. Results

7. Results

Results obtained from the reliability-based calibration of the partial...
safety factors indicated in the main Brazilian codes that guide the design of concrete structures are presented in this section.

7.1 The flexural resistance of reinforced concrete beams

Table 6 presents the set of partial safety factors currently indicated in the actions code [1] and in the concrete structures design code [2], as well as the same set after calibration, related to the flexural resistance of reinforced concrete beams.

It is possible to notice that the reliability-based calibration resulted in a decrease in the values of $\gamma_c$, $\gamma_s$, $\gamma_g$, $\psi_q$ and $\psi_w$, which was compensated by the increase in the values of $\gamma_q$ and $\gamma_w$. Similar to the results observed by Beck and Souza Jr. [4], partial factors resulting from the calibration process increased the primary loads and reduced the secondary loads.

Figure 2 shows the variations of reliability indexes for different loads considering the safety factors before and after calibration. This figure shows that the calibrated factors lead to greater uniformity in reliability for different designs and load combinations, given the reduction in dispersion of results and the increase in the average reliability index from 2.95 to 3.0. In order to allow an evaluation of the influence of concrete strength in the calibration, Figure 3 presents the variation of reliability indexes for the concrete classes considered in this study. It shows that for both sets, there is a greater dispersion in the results at the upper limit. The beams with higher longitudinal reinforcement ratios present greater reliability, yet these beams also present greater dispersion in the results. The behavior illustrated in Figure 3 results from an increase in the height of the neutral line, which is accompanied by an increase in the height of the concrete stress block, raising the relative importance of variable $f_c$, which is directly affected by the differences in its parameters in each concrete class.

The calibrated factors are interesting because they lead to safer concrete beams, but the analysis cannot be limited to technical aspects; it should contemplate at least a brief appreciation of the economic impacts of the new factors.
For $\psi_q = 0.5$ the new factors increase the average load by 2.0%, for $\psi_q = 0.7$ the new factors do not increase the average load, and for $\psi_q = 0.8$ the new factors decrease the average load by 4.0%.

### Table 7

Partial safety factors with and without calibration for shear resistance of reinforced concrete beams

<table>
<thead>
<tr>
<th>Factors</th>
<th>Without calibration</th>
<th>With calibration $\beta_{\text{target}} = 3.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_c$</td>
<td>1.40</td>
<td>1.37 (1.35)*</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>1.15</td>
<td>1.16 (1.15)*</td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>1.40</td>
<td>1.25 (1.25)*</td>
</tr>
<tr>
<td>$\gamma_w$</td>
<td>1.40</td>
<td>1.68 (1.70)*</td>
</tr>
<tr>
<td>$\psi_q$</td>
<td>0.50/0.70/0.80</td>
<td>0.34 (0.35)*</td>
</tr>
<tr>
<td>$\psi_w$</td>
<td>0.60</td>
<td>0.31 (0.30)*</td>
</tr>
<tr>
<td>$\psi_q.\psi_w$</td>
<td>0.70/0.98/1.12</td>
<td>0.57 (0.59)*</td>
</tr>
<tr>
<td>$\psi_w.\psi_w$</td>
<td>0.84</td>
<td>0.50 (0.49)*</td>
</tr>
</tbody>
</table>

*Approximate values in parentheses.

Table 7 presents the set of partial safety factors currently indicated in the actions code [1] and in the concrete structures design code [2], as well as the same set after the calibration, related to the shear resistance of reinforced concrete beams. Again, it is possible to observe that the calibrated factors increase the primary loads and reduce the secondary loads. The difference in the factors calibrated for flexural and shear resistance results from the fact that the same target reliability index was adopted for both problems, whereas each one presents an average reliability index of its own.

Figure 4 illustrates the variations of reliability indexes for different loads, considering the safety factors before and after calibration, while figure 5 presents the variations of reliability indexes for the concrete classes considered in this work. Both figures show that calibrated factors lead to greater uniformity in reliability, for different design cases and load combinations, given the reduction in dispersion of results and the increase in the average reliability index from 2.89 to 3.0.

### Figure 4

Reliability index bounds for shear resistance of reinforced concrete beams and $\beta_{\text{target}} = 3.0$

### Figure 5

Reliability index bounds for shear resistance of reinforced concrete beams, all concrete classes and $\beta_{\text{target}} = 3.0$
In the calibration for shear and for \( \psi = 0.5 \) the new factors increase the average load by 3.0%, for \( \psi = 0.7 \) the new factors increase the average load by 0.5%, and for \( \psi = 0.8 \) the new factors decrease the average load by 1.0%.

### 7.3 Additional considerations

Although the impacts of the new partial safety factors vary from project to project, the sets calibrated in this study are interesting from a technical point of view. The presented results reinforce the importance of a calibration that contemplates other types of structural elements and materials, and a detailed study on the economics impacts of the new factors.

### 8. Conclusions

This paper presented a study on reliability-based calibration of partial safety factors indicated in the main Brazilian codes used in the design of concrete structures. The study considered beams with concrete of five classes (C20, C30, C40, C50 and C60), three ratios between base and effective depth (0.25, 0.50 and 0.75), three longitudinal reinforcement ratios \( (\rho_{\text{min}}, 0.5\% \) and \( \rho_{\text{max}}) \) and three transverse reinforcement ratios \( \left( \frac{A_s}{s} \right)_{\text{min}}, \left( \frac{A_s}{s} \right)_{\text{mea}} \) and \( \left( \frac{A_s}{s} \right)_{\text{max}} \).

The work also considered a single value for the target reliability index \( (\beta_{\text{target}} = 3.0) \), seven ratios between live loads and permanent loads \( (q^0/g^0) \), and seven ratios between wind loads and permanent loads \( (w^0/g^0) \). The study showed that calibrated factors result in greater uniformity in reliability for different design cases and load combinations, which is mainly achieved by increase in principal loads, and reduction in the secondary loads.

### 9. Acknowledgments

The authors are grateful to CNPq and CAPES for the founding, as well as the professionals who provided the data used to characterize Brazilian statistics for concrete and steel strength, as well as dead loads.

### 10. References


